UNIVERSITY OF BIRMINGHAM

Electromagnetism 2
(spring semester 2025)

Lecture 1
Static electric and magnetic fields

- Electrostatic field, electrostatic potential, Gauss law.
- Static magnetic field, Ampere's law
- Lorentz force and relativity

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EM2: practical details

Course materials:

- The course covers three main topics.
  - ✓ **Lectures 1–6:** revision and extension of EM1, leading to **Maxwell's equations in free space**.
  - ✓ **Lectures 7–12:** conductors, dielectric and magnetic media, leading to **Maxwell's equations in media**.
  - ✓ **Remaining lectures: electromagnetic waves** & their properties.
- Materials available on Canvas: lecture notes; five non-assessed problem sheets; past examination papers.

Links to other modules:

- Knowledge of EM1, vector calculus, geometry (e.g. solid angles) is assumed.
- EM2 is essential for many optional Y3/Y4 modules: radiation and relativity; images and communication, physical principles of radar, …

Reading list (from basic to more advanced):

- I.S. Grant, W.R. Phillips: Electromagnetism, 2nd ed. (Wiley)
- R. Feynman, R. Leighton, M. Sands: The Feynman Lectures on Physics (Vol II)
- W.N. Cottingham, D.A. Greenwood: Electricity and Magnetism (Cambridge)
- D.J. Griffiths: Introduction to Electrodynamics, 4th ed. (Cambridge)
- J.D. Jackson: Classical Electrodynamics, 3rd ed. (Wiley)

The system of units

The system of units

- The SI system is used in this course.
- Some textbooks use the Gaussian system.
- The SI is convenient for practical computations but has fundamental drawbacks wrt the Gaussian system.
  - ✓ Conversion factors are introduced: the *electric constant* $\varepsilon_0 \approx 8.85\times10^{-12}$ C/(Vm) and the *magnetic constant* $\mu_0 \approx 4\pi\times10^{-7}$ H/m.
  - ✓ Only their product has a physical meaning ($\varepsilon_0\mu_0 = 1/c^2$).
  - ✓ Different units used for electric (**E**, **D**) and magnetic (**B**, **H**) fields (V/m, C/m^2, T, A/m), though the distinction between electric vs magnetic fields is relative.

Electrostatic field (1)

Coulomb's law (1785): force acting on a point charge **q** in the field of a point charge **Q**.
Very strong wrt gravity! Two protons: $F_G/F_C \sim 10^{-36}$.

$$\vec{F} = \frac{Qq}{4\pi\varepsilon_0} \cdot \frac{\vec{r}}{r^3}$$

Electrostatic field of a point charge Q:

$$\vec{E}(\vec{r}) = \frac{\vec{F}}{q} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{\vec{r}}{r^3}$$

The electric field, introduced formally above, is "real" and has energy, momentum, etc. *[unit: V/m]*

$\vec{E}(\vec{r})$ is a *conservative vector field*: we can define a scalar potential φ,

$$\vec{E} = -\left(\frac{\partial\varphi}{\partial x}\vec{e}_x + \frac{\partial\varphi}{\partial y}\vec{e}_y + \frac{\partial\varphi}{\partial z}\vec{e}_z\right) = -\mathbf{grad}\ \varphi = -\nabla\varphi$$

This also means that $\oint \vec{E}d\vec{l} = 0$ for any closed curve.

> Physical meaning of φ_A in electrostatics: work against the field required to bring a unit test charge from infinity to the point A.

For a point charge Q, $\varphi(r) = \int_r^\infty \frac{Q}{4\pi\varepsilon_0 t^2}dt = -\frac{Q}{4\pi\varepsilon_0 t}\Big|_r^\infty = \frac{Q}{4\pi\varepsilon_0 r}$

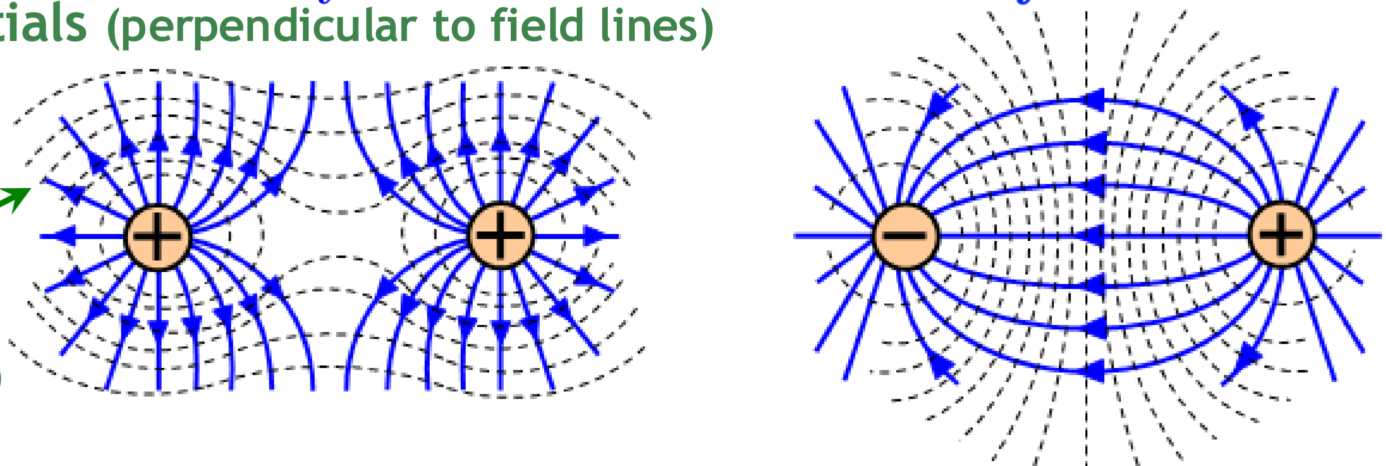
Electrostatic field (2)

The *principle of superposition*: for multiple point changes Q_i,

the total field is $\vec{E} = \sum_i \vec{E}_i$, therefore $\varphi = \sum_i \varphi_i$

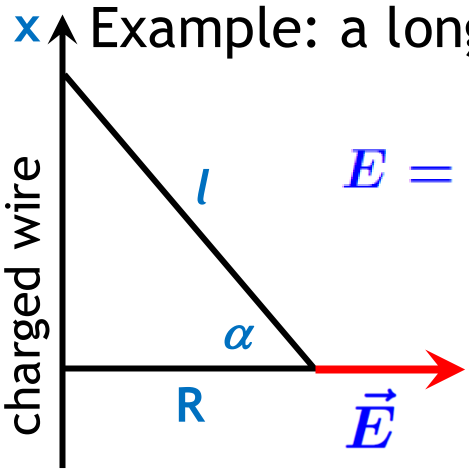
Equipotentials (perpendicular to field lines)

Field lines (tangent to the field vector)



Now we can compute electric field for any *known* charge distribution.

Example: a long wire with a constant linear charge density $\lambda > 0$ [C/m].



$$E = \frac{\lambda}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} \frac{dx}{l^2} \cos\alpha = \frac{\lambda}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} d(R\tan\alpha) \frac{\cos^2\alpha}{R^2} \cos\alpha$$

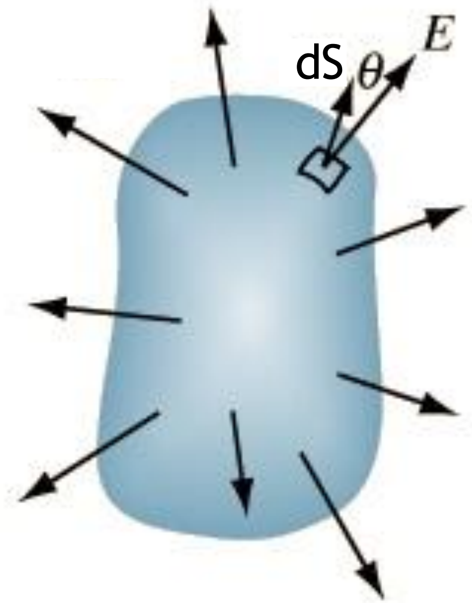
$$= \frac{\lambda}{4\pi\varepsilon_0 R} \int_{-\pi/2}^{+\pi/2} \cos\alpha\, d\alpha = \frac{\lambda}{2\pi\varepsilon_0 R}$$

Gauss law

Gauss law valid for the electric field
(universally valid, also for non-static fields):

$$\int_S \vec{E} d\vec{S} = \frac{1}{\varepsilon_0} \int_V \rho dV$$

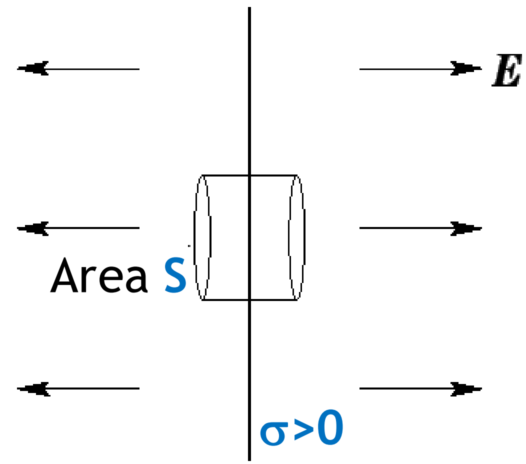
Electric flux through any closed surface — Total electric charge within the surface



- Gauss law is *equivalent to Coulomb's law*.
- Gauss law simplifies field computations for systems with a sufficient degree of symmetry.
- Gauss law is not sufficient for computations in general case: a single scalar relation for the three unknowns E_x, E_y, E_z.
- Similarity of Coulomb's law and Newton's law of gravity ($F \sim 1/r^2$): Gauss law applies also to the gravitational field.

Gauss law: examples

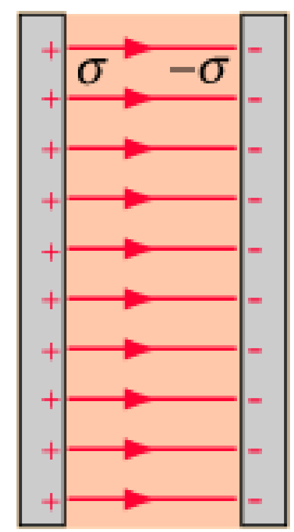
1) Infinite plane, surface charge density σ [C/m^2].
Symmetry: field is perpendicular to the plate.



Gauss law: $2SE = \sigma S/\varepsilon_0$; $E = \dfrac{\sigma}{2\varepsilon_0}$

2) For a planar capacitor, use superposition:

$E = \dfrac{\sigma}{\varepsilon_0}$ between the plates; $E = 0$ outside.



3) Uniformly charged solid sphere (radius R, charge Q).
Inside the sphere,

$$4\pi r^2 E = \frac{Q}{\varepsilon_0}\left(\frac{r}{R}\right)^3 \qquad \boxed{E(r) = \frac{Qr}{4\pi\varepsilon_0 R^3}}$$

Outside the sphere,

$$4\pi r^2 E = \frac{Q}{\varepsilon_0} \qquad \boxed{E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

Static magnetic field

The *current density* vector: $\vec{j} = \rho\vec{v}$ [A/m^2]

Right-hand side: product of charge density [C/m^3] and velocity [m/s].

Biot–Savart law (1820): magnetic field due to a *linear elementary current*, or a *volume elementary current*:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\cdot\frac{Id\vec{l}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\cdot\frac{(\vec{j}\times\vec{r})dV}{r^3}$$ [unit: T]

Magnetic force acting on an elementary current **I·dl** or **j·dV**:

$$\vec{F} = Id\vec{l}\times\vec{B} = (\vec{j}\times\vec{B})dV$$

Equivalently, *Lorentz force* acting on a charge **q** moving in a magnetic field with velocity **v**:

$$\vec{F} = q\vec{v}\times\vec{B}$$

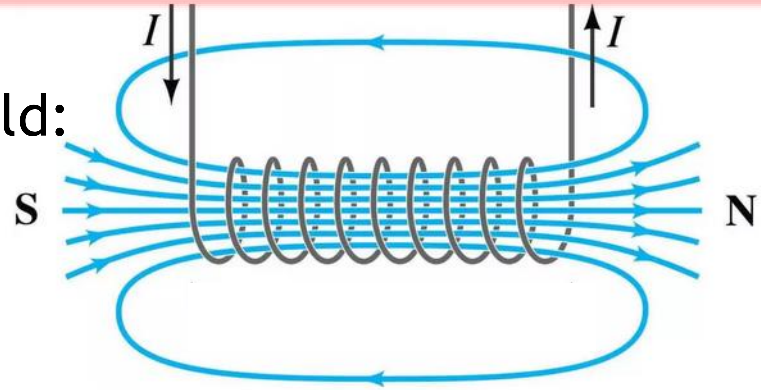
The principle of superposition: $\vec{B} = \sum_i \vec{B}_i$

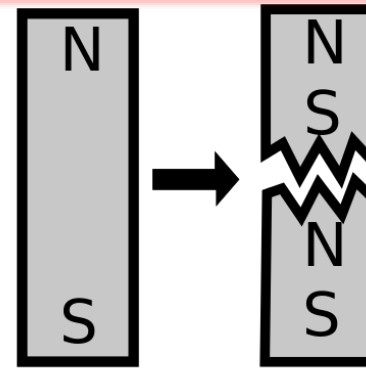
Example: attraction (repulsion) of parallel (antiparallel) currents.

Magnetic field lines; Ampere's law

Continuous field lines, no sources of the **B** field:

$$\int_S \vec{B} d\vec{S} = 0$$



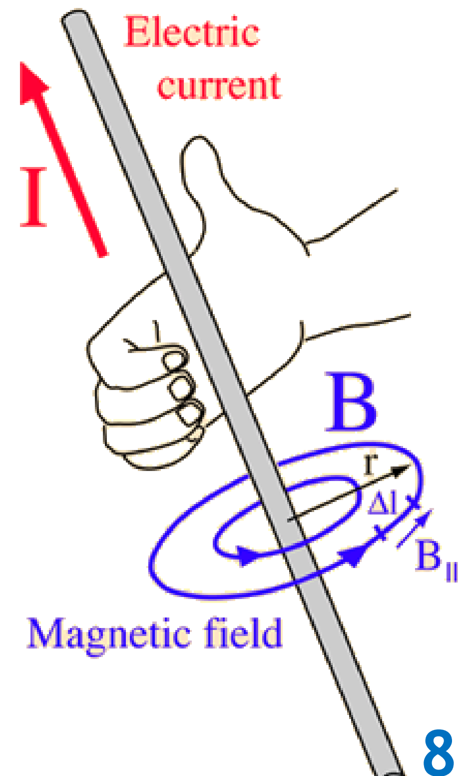


Ampere's law for the magnetic field produced by steady currents:

$$\oint_L \vec{B} d\vec{l} = \mu_0 \int_S \vec{j} d\vec{S}$$

Line integral of the magnetic field around a closed curve

Current through *any* surface enclosed by the curve

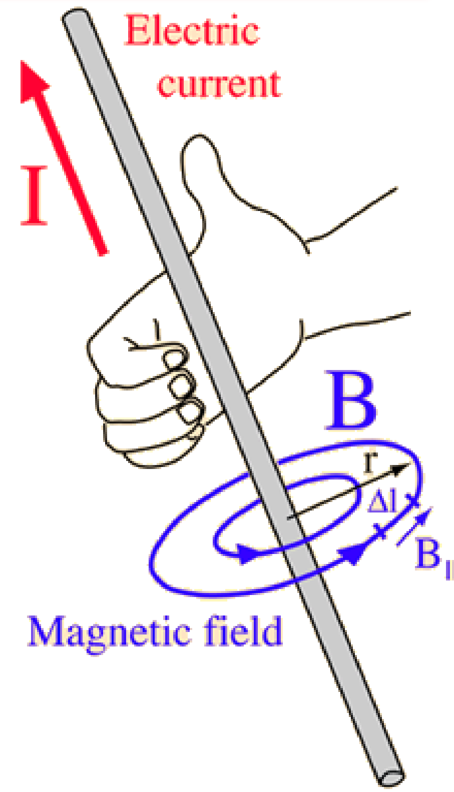


Sign convention: the direction of **B** relative to **j** is determined by the *right-hand rule*.

Ampere's law: examples

1) Straight infinite wire: cylindrical symmetry.
Outside the wire,

$$2\pi r B = \mu_0 I \qquad B = \frac{\mu_0 I}{2\pi r}$$

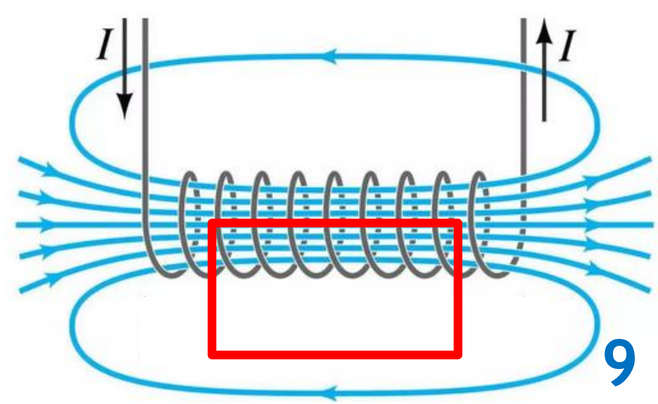


For I = 1 A and r = 2 cm,
$B = 4\pi\times10^{-7}\ \mathrm{H{\cdot}m^{-1}}\cdot 1\mathrm{A} / (2\pi\cdot 0.02\ \mathrm{m}) = 10^{-5}\ \mathrm{T}$
(comparable to the Earth's magnetic field)

Inside the wire, assuming uniform current density **j**,

$$2\pi r B = \mu_0 I (r/R)^2 \qquad B = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 j r}{2}$$

2) Long solenoid with **N** turns per unit length.
Outside, **B=0** by considering it part of a very large toroid and using cylindrical symmetry.



Inside the solenoid,

$$BL = \mu_0 \cdot NL \cdot I \qquad B = \mu_0 N I$$

Lorentz force and relativity

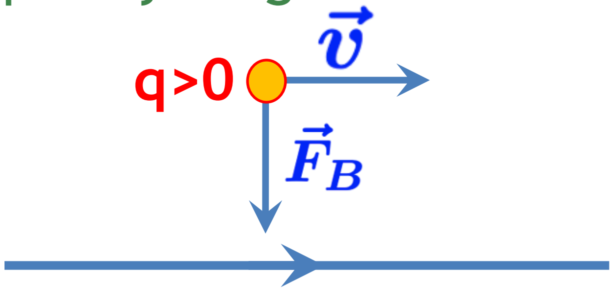
[not discussed in the lecture]

❖ Lorentz force acting on a point charge **q** in **E** and **B** fields:

$$\vec{F} = \vec{F}_E + \vec{F}_B = q(\vec{E} + \vec{v} \times \vec{B})$$

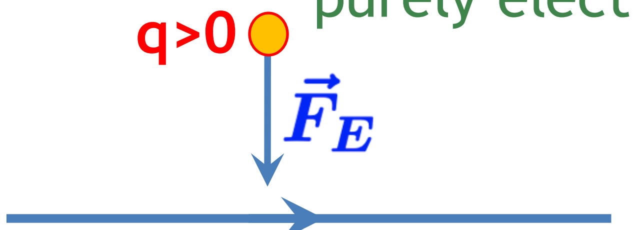
❖ Force on a charge moving along a wire carrying a current:

Lab reference frame S:
purely magnetic force



Current **I**, charge density **λ=0**

Reference frame of the charge S':
v=0, F_B=0, therefore the force is purely electric



Current **I'**, charge density **λ'≠0 !**

❖ The wire becomes charged (**λ'≠0**) in the reference frame **S'**! This is a *relativistic length contraction* effect.

❖ Electric and magnetic forces are two manifestations of the same phenomenon, *electromagnetism*.

❖ Electromagnetism is intrinsically *relativistic*.

Summary

❖ Laws of electrostatics and magnetostatics *in free space*, *in the integral form*, useful for practical computations:

Gauss law (*universally valid*)

$$\int_S \vec{E} d\vec{S} = \frac{1}{\varepsilon_0} \int_V \rho dV$$

Conservative nature of the **E**-field (*static field only*)

$$\oint_L \vec{E} d\vec{l} = 0$$

Absence of magnetic poles (*universally valid*)

$$\int_S \vec{B} d\vec{S} = 0$$

Ampere's law (*static field only*)

$$\oint_L \vec{B} d\vec{l} = \mu_0 \int_S \vec{j} d\vec{S}$$