UNIVERSITY OF BIRMINGHAM

Electromagnetism 2
(spring semester 2025)

Lecture 5
Magnetic dipoles; EM induction

- Magnetic dipoles and their properties
- Faraday's law of electromagnetic induction
- Electromotive force in a loop of wire

Prof. Evgueni Goudzovski • Physics West 212 • goudzovs@cern.ch

Previous lecture

❖ Electrostatic potential of a point-like electric dipole:

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\vec{r}}{r^3}$$

❖ Electric dipoles in uniform fields: zero net force; non-zero torque; dipole moments tend to align along the field lines.

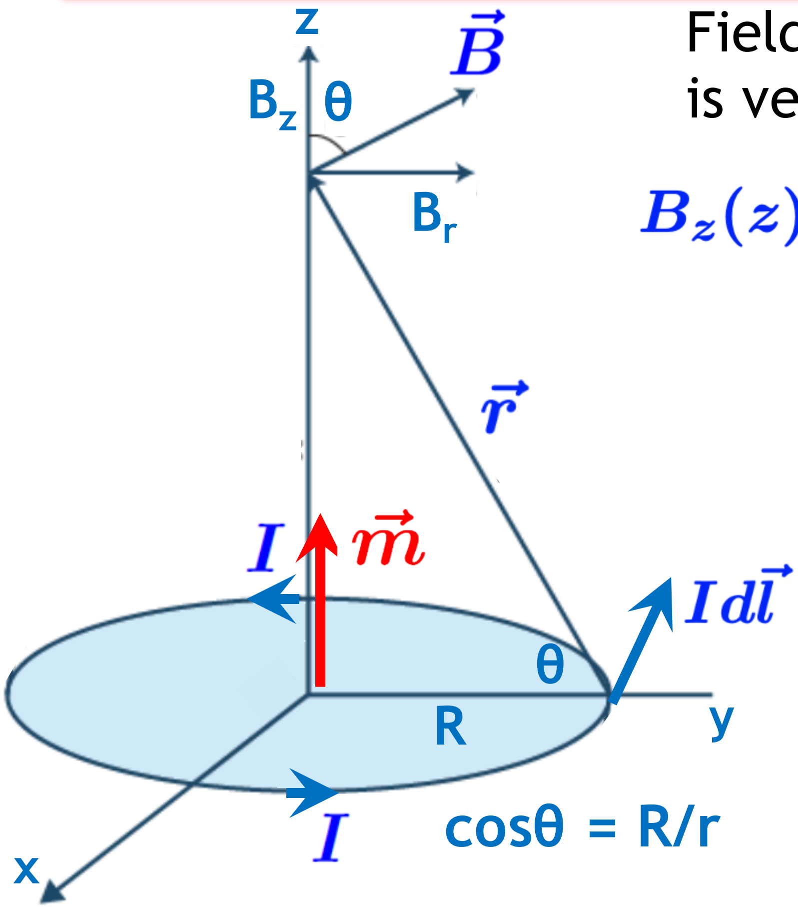
$$\vec{\tau} = \vec{p} \times \vec{E} \qquad U = -\vec{p} \cdot \vec{E}$$

❖ Electric dipoles in non-uniform fields: non-zero net force. A dipole aligned with the field is attracted towards stronger field.

$$\vec{F} = (\vec{p}\nabla)\vec{E}$$

❖ Electrically neutral objects with non-zero dipole moments (**Q=0**, **p≠0**), e.g. water molecules, experience electrostatic forces.

Magnetic dipole



Field of a circular current loop on the z axis is vertical by symmetry,

$$B_z(z) = \frac{\mu_0}{4\pi}\oint_L \frac{Idl}{r^2}\cos\theta = \frac{\mu_0 I\cos\theta \cdot 2\pi R}{4\pi r^2}$$

$$= \frac{\mu_0 I \cdot \pi R^2}{2\pi r^3} = \frac{\mu_0 m}{2\pi r^3} \xrightarrow{|z| \gg R} \frac{\mu_0 m}{2\pi |z|^3}$$

Identical to the electric dipole field,

$$E_z(z) = \frac{p}{2\pi\varepsilon_0 |z|^3} \quad \textit{(lecture 4)}$$

with a substitution $1/\varepsilon_0 \to \mu_0$.

Magnetic dipole moment:

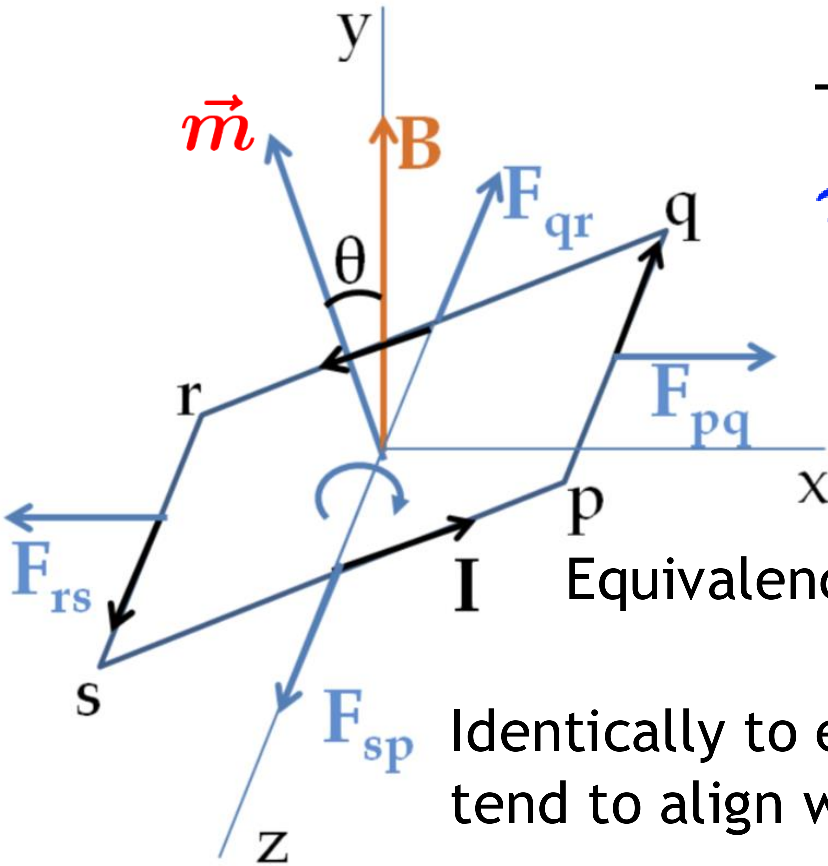
$$\vec{m} = I\vec{S}$$

More generally, fields of point-like magnetic and electric dipoles are identical at all points in space.

A planar *current loop* is equivalent to a *pair of magnetic charges*.
A current loop, an electron or a hydrogen atom are magnets.

Magnetic dipole in uniform field

A square current loop (area S=a×a) in a *uniform* external B field



No net force acting on the loop:

$$\vec{F} = \oint_L I d\vec{l} \times \vec{B} = -I\vec{B} \times \oint_L d\vec{l} = \vec{0}$$

Torque exerted on a loop:

$$\tau = Fa \sin\theta = IBa \cdot a \sin\theta$$
$$= ISB \sin\theta$$

Using the magnetic dipole moment, in the vector form, $\vec{\tau} = \vec{m} \times \vec{B}$

Equivalence to electric dipoles: $\vec{\tau} = \vec{p} \times \vec{E}$
(lecture 4)

Identically to electric dipoles, magnetic dipoles tend to align with the field; the potential energy is

$$U = -\vec{m} \cdot \vec{B}$$

Example: a compass needle.

Dipole in non-uniform field

The analogy extends to *non-uniform* external fields.

Force acting on an electric dipole:
(lecture 4)

$$\vec{F} = (\vec{p}\nabla)\vec{E}$$

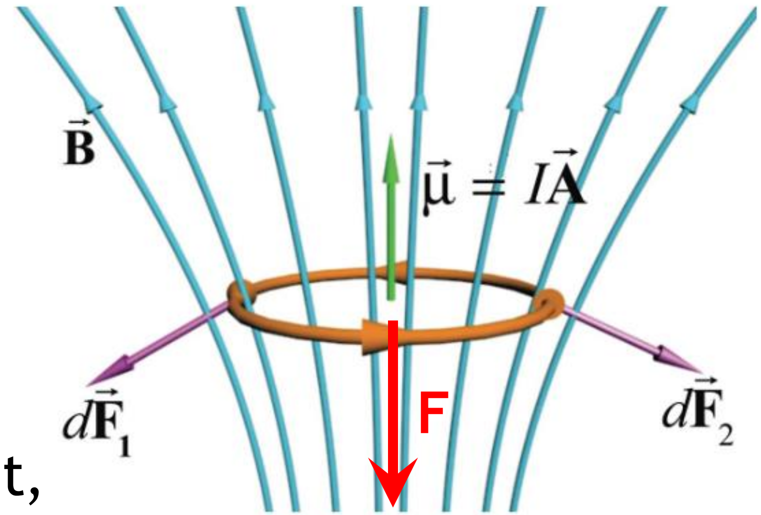
Analogously, force on a magnetic dipole:

$$\vec{F} = (\vec{m}\nabla)\vec{B}$$

In case both the dipole moment and the magnetic field only have the **x** component,

$$F_x = m_x \frac{\partial B_x}{\partial x}$$

Weaker **B** field

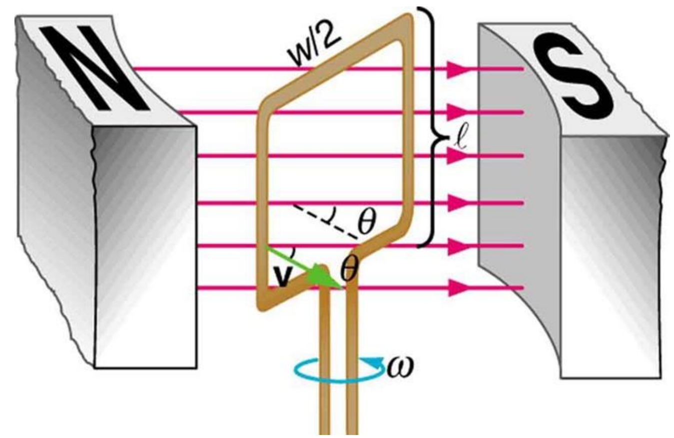


Stronger **B** field

- A dipole tends to align with the external field.
- Then the force is directed towards the region of stronger field.
- This leads to attraction of parallel electric dipoles and parallel magnetic dipoles (including permanent magnets).

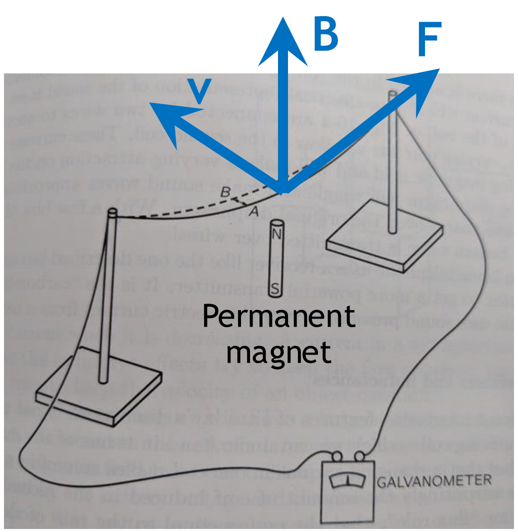
Induced currents

Electric currents, and magnets, produce magnetic fields



- **B**-field produces Lorentz force on moving charges: $\vec{F} = q\vec{v} \times \vec{B}$
- Applications: AC generators and motors; measurements of currents and **B**-fields.
- There are magnetic forces between currents.

Do magnets produce electric fields or currents?



- Transverse movement of the *wire*: a pulse of current observed, due to the Lorentz force.
- Transverse movement of the *magnet*: no Lorentz force but *a pulse of current is still observed*.
- This is a new phenomenon: *induced current* due to the variation of the magnetic field.

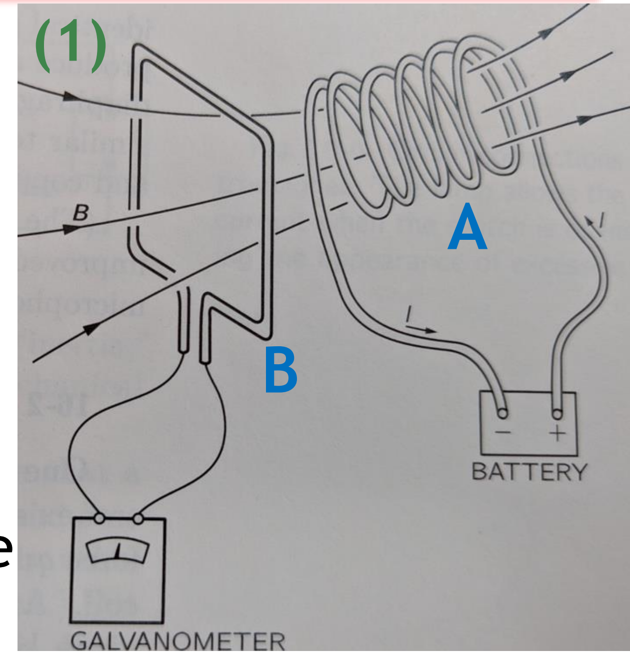
Sketches borrowed from the Feynman lectures on physics

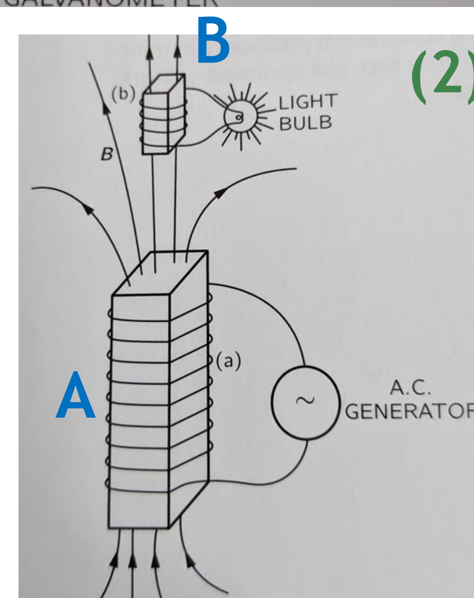
Examples of induced currents

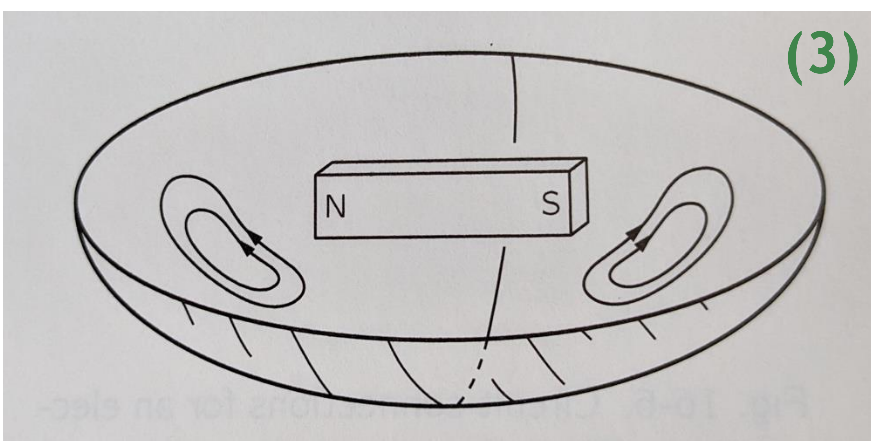
(1) Replacing a permanent magnet with a coil: a pulse of current is observed in loop **B** due to *change of current, position or orientation* of the coil **A**.

(2) Alternating current is induced in coil **B** by an alternating current in coil **A**.

(3) Magnetic levitation: a magnet placed above superconductor surface is repelled by the **B**-field created by the induced *eddy current*.







Sketches borrowed from the Feynman lectures on physics

Faraday's law

Faraday's law for the *induced electric field*, in the differential form:

(universally valid)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Unlike the electrostatic field *(lecture 1)*, the **electric field** is **not conservative** in the general case

Using the curl theorem, for *any* loop L and any surface enclosed,

(lecture 2)

$$\oint_L \vec{E} d\vec{l} = \int_S (\nabla \times \vec{E}) d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{S}$$

Faraday's law in the integral form:

$$\oint_L \vec{E} d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{S}$$

Line integral of the induced electric field around a closed curve

Rate of change of magnetic flux through *any* surface enclosed by the curve

Lenz's law (the minus sign): the direction of the induced current (if one were to flow) is such that its effect would oppose the change in magnetic flux.

Electromotive force in a loop

❖ Electromotive force (*EMF*) in a circuit: $\boldsymbol{\mathcal{E} = \frac{1}{q}\oint_L \vec{F}_{\mathrm{Lorentz}} d\vec{l}}$

❖ Two ways of inducing EMF in a loop of wire, due to the Lorentz force acting on a charge **q**:

$$\vec{F}_{\mathrm{Lorentz}} = q(\underbrace{\vec{E}} + \underbrace{\vec{v} \times \vec{B}})$$

by electromagnetic induction, i.e. by variation of the magnetic field (Faraday's law)

by motion of the wire (deformation or change of orientation, **v≠0**); known as *motional EMF*

❖ The two phenomena are distinct and independent. However, both lead to the same formula for the EMF:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

(rate of change of the magnetic flux through the loop)

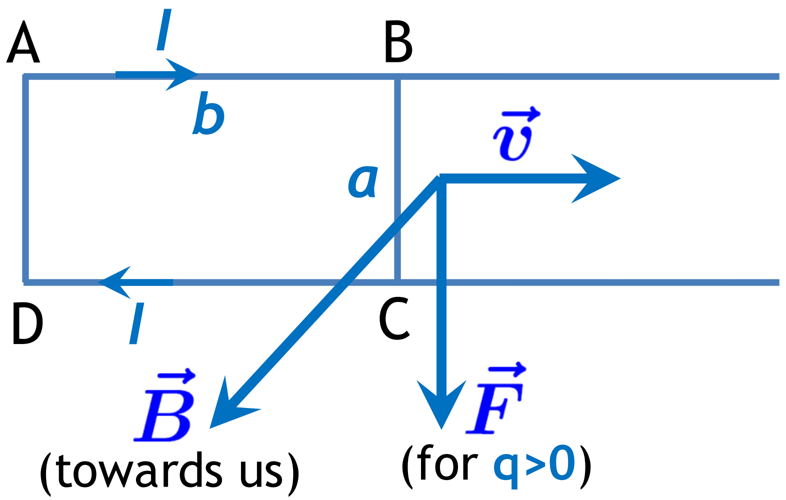
Proof of the EMF formula

Via electromagnetic induction:

The formula follows directly from Faraday's law,

$$\mathcal{E} = \frac{1}{q}\oint_L \vec{F}_{\text{Lorentz}} d\vec{l} = \frac{1}{q}\oint_L q\vec{E}d\vec{l} = \underbrace{\oint_L \vec{E}d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{S}}_{\text{Faraday's law}}$$

Via deformation of the wire:



A fixed U-shaped loop;
a crossbar (BC) moving with a speed v;
uniform static external B-field.

Lorentz force: $\vec{F} = q\vec{v} \times \vec{B}$

Motional EMF:

$$|\mathcal{E}| = \frac{1}{q}\int_B^C \vec{F}d\vec{l} = \frac{1}{q}\cdot qvBa = Ba\frac{db}{dt} = \frac{d(abB)}{dt} = \frac{d\Phi_B}{dt}$$

Example: dipole field and EMF

[not discussed in the lecture]

EMF in a loop of wire L due to varying magnetic field of a moving magnet:

For L≫r, the point-like dipole approximation is valid.

Magnetic flux through the loop:

$$\Phi_B \approx \frac{\mu_0 m}{2\pi L^3} \cdot \pi r^2 = \frac{\mu_0 m r^2}{2L^3}$$

Magnitude of the electromotive force:

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dL} \cdot \frac{dL}{dt} = -\frac{3\mu_0 m r^2}{2L^4} \cdot (-v) = \frac{3\mu_0 m v r^2}{2L^4}$$

Summary

- Our current (still incomplete) understanding of the equations of electromagnetism *in free space*:

Gauss law (*universally valid*)

$$\nabla \vec{E} = \rho/\varepsilon_0$$

Faraday's law (*universally valid*)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Absence of magnetic poles (*universally valid*)

$$\nabla \vec{B} = 0$$

Ampere's law (*static field only*)

$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

- Variation of magnetic field gives rise to electric field.
- Does variation of electric field give rise to magnetic field?