UNIVERSITY OF BIRMINGHAM

Electromagnetism 2
(spring semester 2025)

Lecture 12
Ferromagnetism; magnets

- ❖ Ferromagnetism; hysteresis
- ❖ Magnetic cores; electromagnets
- ❖ Permanent magnets

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Previous lecture

- For dia- and paramagnetic materials, *magnetic susceptibility* χ_M and *relative permeability* μ are defined as

$$\vec{B} = \mu_0(1 + \chi_M)\vec{H} = \mu_0\mu\vec{H}$$

- Boundary conditions for the **B** and **H** fields:

$$B_{1n} = B_{2n} \quad \text{and} \quad H_{1t} = H_{2t}$$

- Energy density of the magnetic field in dia- and paramagnetics:

$$w = \frac{1}{2}BH$$

Ferromagnetism

- Ferromagnetism: the spins of conduction electrons in some metals *align spontaneously* due to the quantum-mechanical *exchange interaction*. This leads to the formation of *domains*.

- When domains are *not* oriented at random, this leads to a large magnetisation (also in the absence of external field).

- Spontaneous alignment breaks down above a critical temperature, the *Curie point* T_C (for iron, T_C=1043 K).

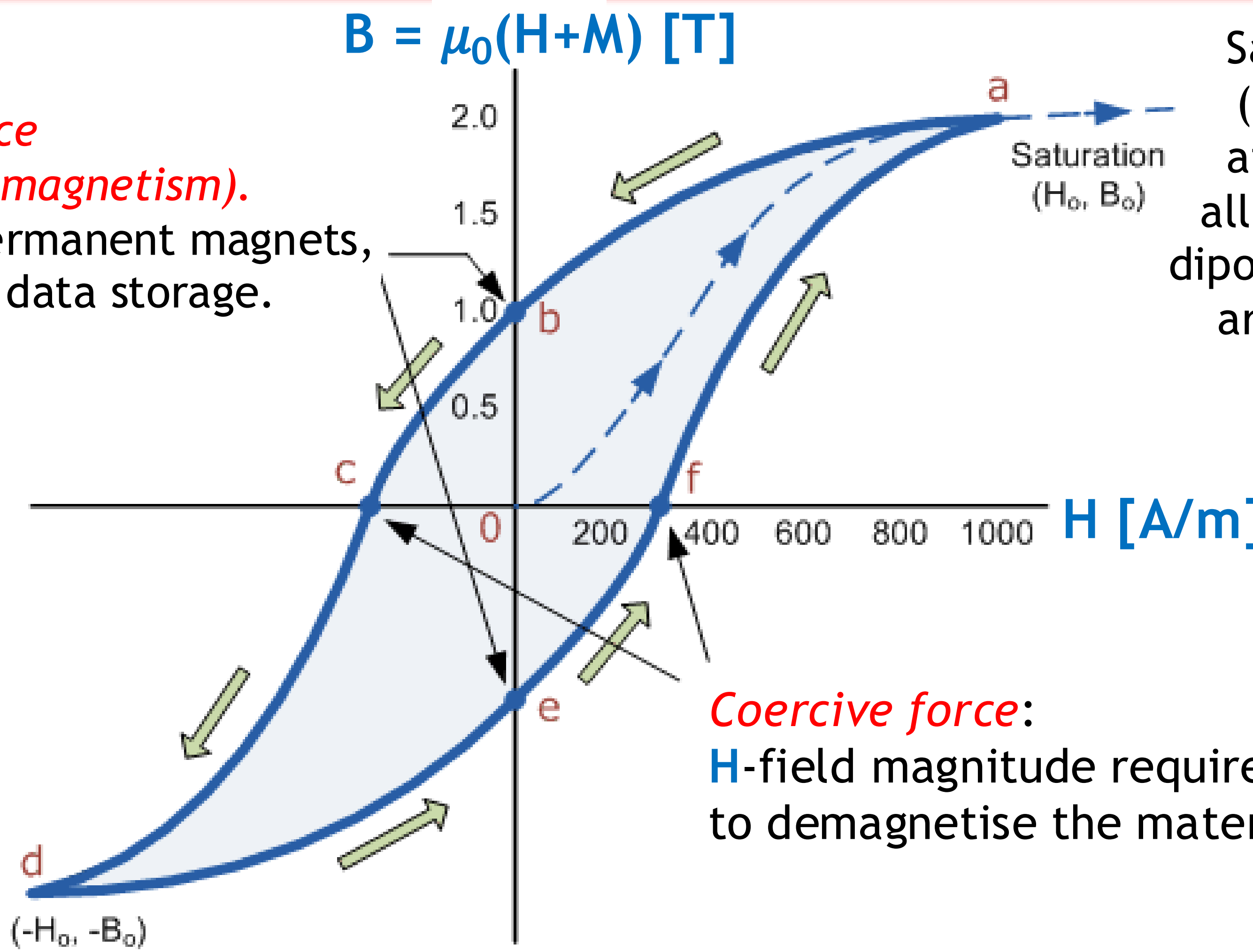
- For $T>T_C$, the material is paramagnetic with $\chi_M = \frac{C}{T - T_C}$ (the *Curie–Weiss law*).

- When all the spins are aligned, the magnetisation is *saturated*. (e.g. maximum B field for steel is about 1.5 T).

- Unlike dia- and paramagnetics, non-linear relation between B and H. Therefore, relative permeability $\mu = B/(\mu_0 H)$ is not well defined.

Hysteresis (or “B–H”) curve

Remanence (residual magnetism).
Usage: permanent magnets, magnetic data storage.



Saturation (typically at B≈1 T): all individual dipole moments are aligned

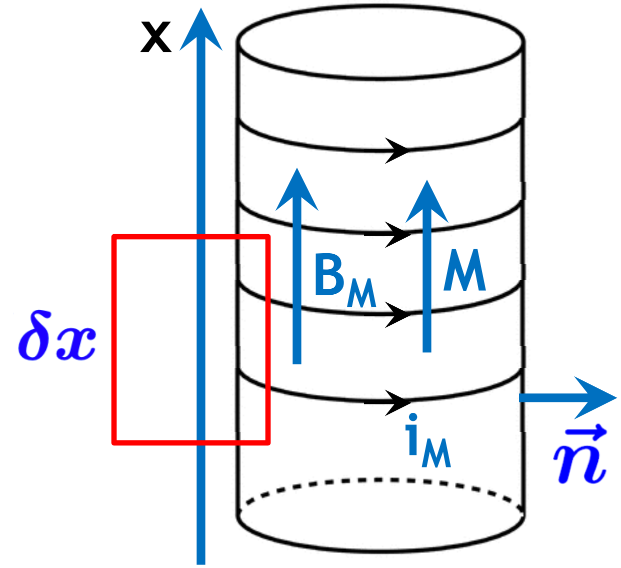
Coercive force:
H-field magnitude required to demagnetise the material.

Indicative relative permeability can be very large:
at the point (H=500 A/m, B=1 T), we obtain $\mu = B/(\mu_0 H) = 1600$.

Saturation field

Ampere's law for the magnetic field **B** due to magnetisation (red contour):

$$B\delta x = \mu_0 i_M \delta x = \mu_0 M \delta x$$



Saturation: all atomic dipole moments, each of the order of Bohr magneton (μ_B=9.27×10^{-24} A·m), are aligned.

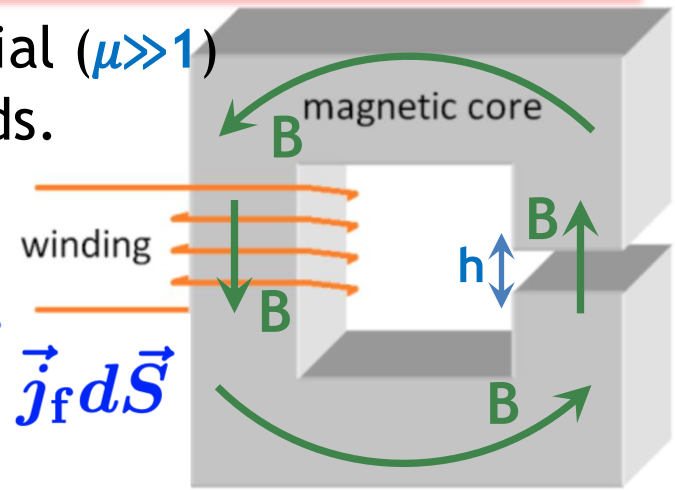
Saturation field:

$$B_{\mathrm{sat}} = \mu_0 M \approx \mu_0 \mu_B n_{\mathrm{atoms}} = \mu_0 \mu_B \rho \frac{N_A}{\mathcal{M}}$$

- n_{atoms}: density of atoms [m^{-3}];
- ρ: density of material [kg/m^3];
- $N_A = 6\times10^{23}$: Avogadro constant;
- $\mathcal{M}$: molar mass [kg/mol].

For iron (ρ=7.9 g/cm^3, $\mathcal{M}$=56 g/mol), we estimate $B_{\mathrm{sat}} \approx$ 1 T. Saturation fields range from 0.2 T to 2 T.

Electromagnets (1)



Magnetic core: a piece ferromagnetic material ($\mu \gg 1$) designed to confine and guide magnetic fields.

Characteristic core length: L.
Gap width: $h \ll L$; N turns of wire; current I.

Ampere's law for the H field: $\oint_L \vec{H} d\vec{l} = \int_S \vec{j}_f d\vec{S}$

Boundary condition $H_{t1}=H_{t2}$, i.e. $B_{out}=B_{in}/\mu \ll B_{in}$: the B-field is contained in the magnetic core.

Boundary condition $B_{n1}=B_{n2}$: same B-field in the core and in the gap.

Assuming a fixed μ value, $\dfrac{BL}{\mu_0 \mu} + \dfrac{Bh}{\mu_0} = NI$

For a small gap ($h \ll L/\mu$), $B = \dfrac{\mu_0 NI}{L/\mu + h} \gg \boxed{\dfrac{\mu_0 NI}{L+h}}$

Magnetic field in the absence of core

The B-field is strongly enhanced by the presence of the core.

Electromagnets (2)

Let's account for hysteresis (not assumption of a fixed μ value).

Ampere's law: $$HL + \frac{Bh}{\mu_0} = NI$$

(**H**: field in the core; **B**: field both in the core and in the gap).

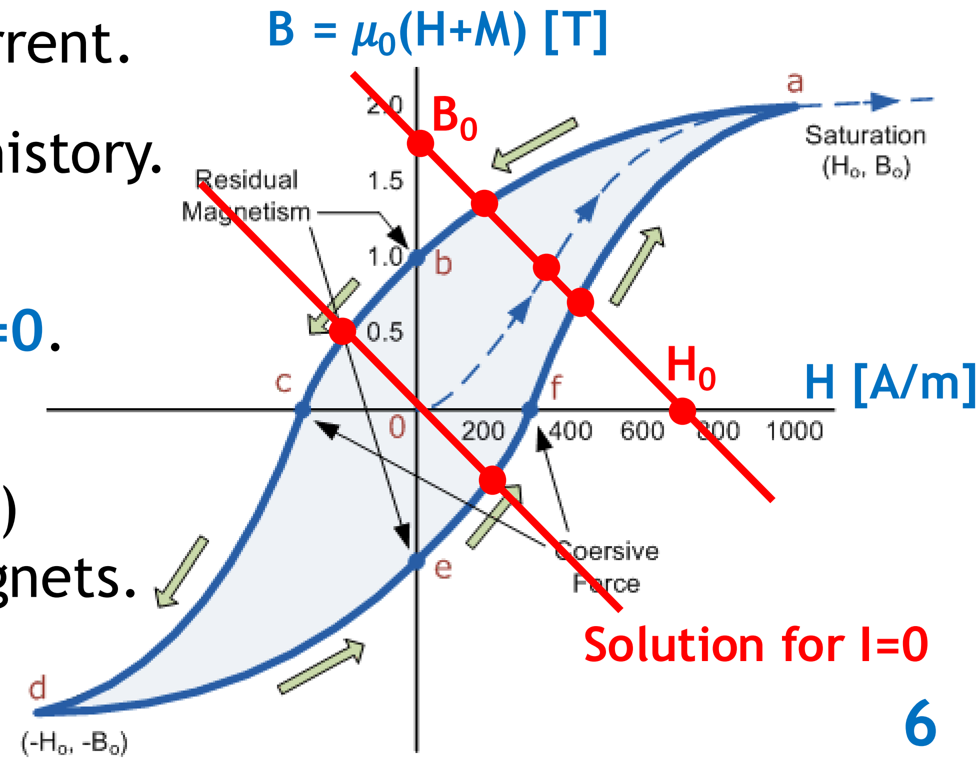
A straight line in the **B–H** plot: axis intercepts $H_0 = NI/L$; $B_0 = \mu_0 NI/h$.

The line shifts when changing current.

Several solutions, depending on history.

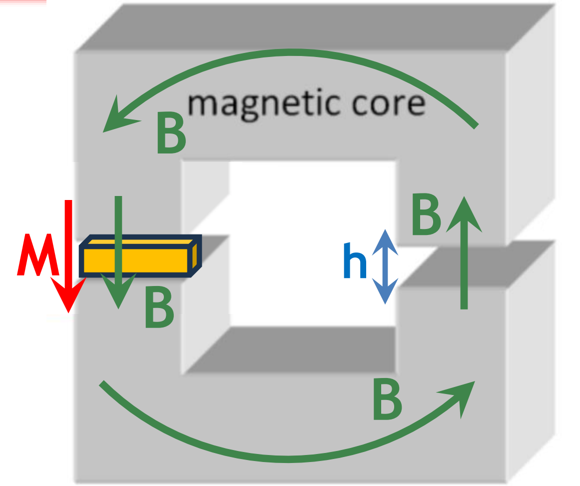
If the core is first saturated,
a considerable residual field at $I=0$.

"Hard materials" with a wide hysteresis loop (e.g. alnico alloys) are used to make permanent magnets.



Magnetic cores: an example

Magnetic core of characteristic length L,
with two gaps (each of width h).



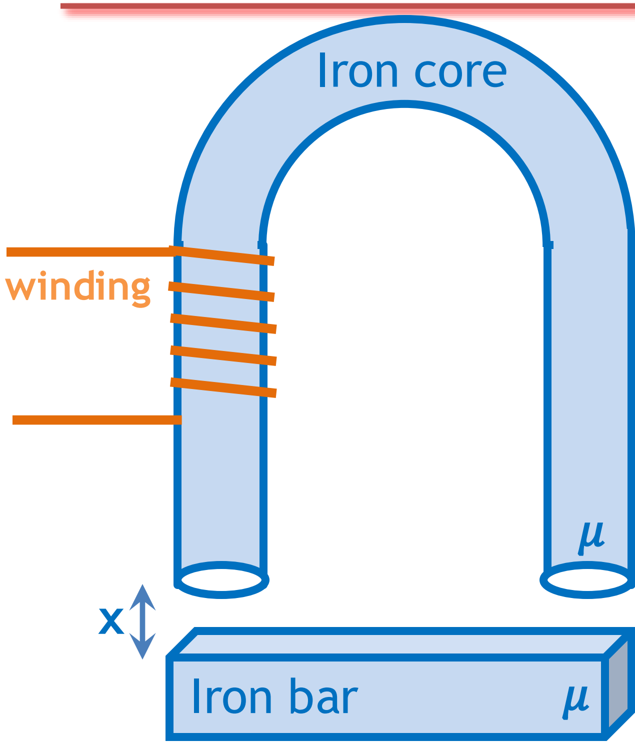
A piece of non-linear magnetised material
with a magnetisation M is inserted into a gap.

Ampere's law for the H field: $$\oint_L \vec{H} d\vec{l} = \int_S \vec{j}_f d\vec{S} = 0$$

$$\frac{BL}{\mu_0 \mu} + \frac{Bh}{\mu_0} + \left(\frac{B}{\mu_0} - M\right) h = 0$$

$$B = \frac{\mu_0 M h}{L/\mu + 2h}$$

Lifting electromagnets



For a process at **B=const**, i.e. Φ_B**=const**, **EDM = −dΦ_B/dt = 0**; therefore energy conservation: **dW_m + dA_{mech} = 0**.

Magnetic field energy:

$$W_m = \frac{B^2}{2\mu_0\mu} V_{\text{iron}} + \frac{B^2}{2\mu_0} Ax$$

A: total area of the two gaps.

Attractive force acting on the bar:

$$F_x = \left(\frac{\partial A_{\text{mech}}}{\partial x}\right)\Bigg|_{\Phi_B} = -\left(\frac{\partial W_m}{\partial x}\right)\Bigg|_{\Phi_B} = -\frac{B^2 A}{2\mu_0}$$

Lifting force per unit area: $f = \frac{|F|}{A} = \frac{B^2}{2\mu_0} = \boxed{w_m}$ Magnetic energy density in the gap

Example: for **B=1 T** (typical saturation field),
$f = B^2/2\mu_0 = (1 / (2\times1.26\times10^{-6}))\ \text{N/m}^2 = 400\ \text{kN/m}^2$.

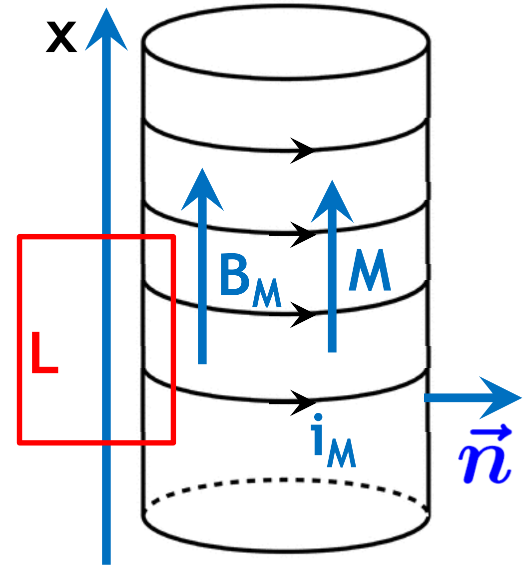
Permanent magnets (1)

For an *infinitely long* magnetised cylinder,
Ampere's law for the B-field inside (red contour):

$$BL = \mu_0 i_M L = \mu_0 M L$$

$$\vec{B} = \mu_0 i_M = \mu_0 \vec{M}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} = 0$$



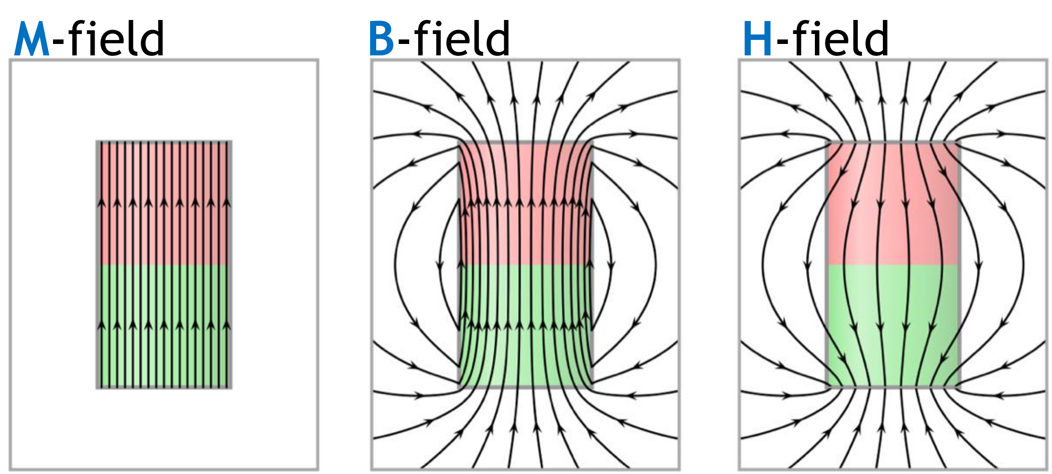
Outside the cylinder, M=B=H=0.

Consider a cylinder of a *finite length* L (such that R≪L).
Slightly lower B field, therefore, H is antiparallel to B and M.

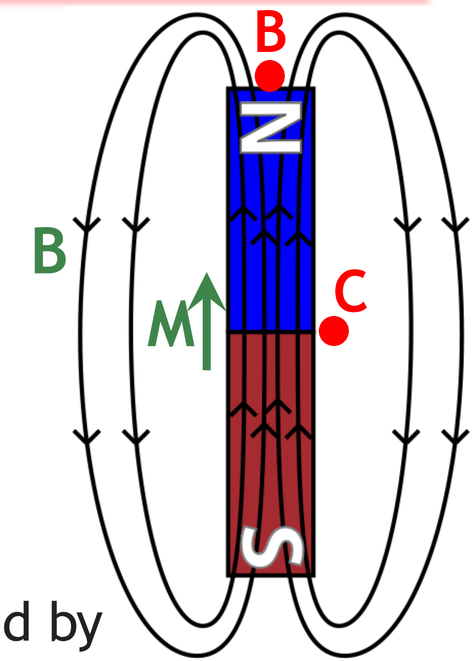
Outside the cylinder,
M=0 (no medium), $H=B/\mu_0$.

B field lines are continuous due to $\nabla\vec{B} = 0$.

The H-field changes direction at the top & bottom surfaces.



Permanent magnets (2)



A uniformly magnetised cylindrical rod (length 2L, radius R≪L).

B-field at the centre A of magnet (see previous page):

$$B_A \approx \mu_0 i_M = \mu_0 M$$

More precisely,

$$B_A = \mu_0 M \frac{\Omega_A}{4\pi} = \mu_0 M \cos\theta \approx \mu_0 M \left(1 - \frac{\theta^2}{2}\right)$$

$$\approx \mu_0 M \left(1 - \frac{R^2}{2L^2}\right)$$

[Ω_A is the solid angle subtended by the magnet surface, and θ=arctan(R/L)]

At point B, by symmetry, $B_B \approx B_A/2 \approx \mu_0 M/2$

At point C, using the boundary condition for H_t,

$$B_C = \mu_0 H_C = \mu_0 H_A = B_A - \mu_0 M = -\frac{R^2}{2L^2}\mu_0 M$$

$|B_C|/|B_B| = (R/L)^2 \ll 1$

On the axis of symmetry at z≫L, dipole field: $B(z) = \dfrac{\mu_0 M V}{2\pi z^3}$

($V = \pi R^2 L$: volume of the magnet)

Summary

- Ferromagnetics: non-linear relation of **B** and **H** fields; relative permeability $\mu = \mathbf{B}/(\mu_0\mathbf{H})$ is not well defined. In practice, often very large effective values ($\mu \sim 10^3$).

- Practical use: magnets. Examples considered: electromagnets, permanent magnets.