Y2 Electromagnetism 2

Non-assessed problem sheet 1 (weeks 1–2)

Revision – Ideal conductors – Electric dipoles

UNIVERSITY OF BIRMINGHAM
School of Physics and Astronomy

1) The electric field near the surface of the Earth has a magnitude of 130 V/m, and is directed downwards. Find the total electric charge of the Earth.

2) Consider a thin wire arranged in a circle of radius R, with a total electric charge Q distributed uniformly. Using Coulomb's law, find the direction and magnitude of the electric field on the axis of symmetry of the wire perpendicular to its plane, as a function of the distance z from the plane. At which points along the axis is the field strongest? Find the asymptotic behaviour of the field for $|z| \gg R$.

3) Consider a thin disc of radius R, with a uniform surface electric charge density σ. Find the electric field on the axis of symmetry of the disc perpendicular to the plane of the disc, at a distance z from the disc. Examine the special cases $|z| \gg R$ and $|z| \ll R$.

4) Consider a solid sphere of radius R with a non-zero volume density of electric charge. At all points inside the sphere, the electric field $\vec{E}$ is directed radially outwards and has the same magnitude E_0. Find the density of electric charge $\rho(\vec{r})$ at each point inside the sphere. Find the total electric charge of the sphere.

5) Consider a circular current loop of radius R carrying an electric current I. Using the Biot–Savart law, find the the direction and magnitude of the magnetic field on the axis of symmetry of the loop perpendicular to its plane, as a function of distance z from the plane. At which points along the axis is the field strongest? Find the asymptotic behaviour of the field for $|z| \gg R$.

6) A closed loop of thin wire carrying a steady current I is placed into a uniform external magnetic field. Prove that there is no magnetic force acting on the wire.

7) Two long parallel wires are placed 1 cm apart and carry electric currents of 10 A in the same direction. Do the wires attract or repel? Find the magnetic force per unit length.

8) Recall the definitions of the del, Laplace, gradient, divergence and curl operators. Prove the following identities in 3-dimensional space, using Cartesian coordinates:

$$\nabla\vec{r} = 3, \quad \nabla^2(r^2) = 6, \quad \nabla(r^n) = nr^{n-1}\vec{\hat{r}},$$

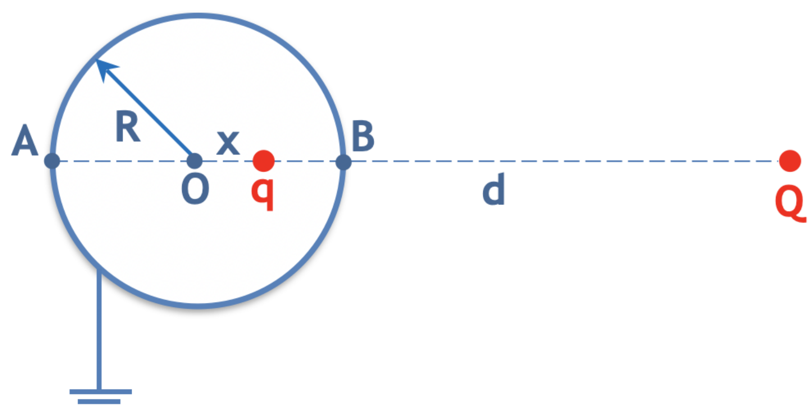
$$\nabla(\vec{r}/r^3) = 0, \quad \nabla\times(\vec{r}/r^3) = \vec{0}, \quad \nabla(\vec{a}\vec{r}) = (\vec{a}\,\nabla)\vec{r} = \vec{a}.$$

Here $\vec{r} = \vec{e}_x x + \vec{e}_y y + \vec{e}_z z$ is the radius vector, and $\vec{\hat{r}} = \vec{r}/r$ is the unit radial vector.

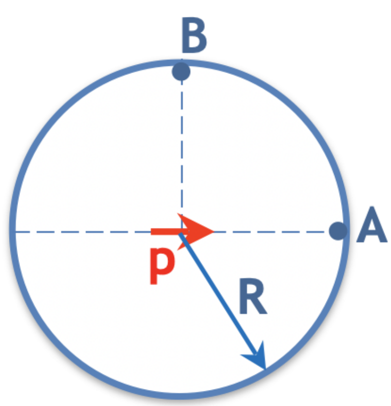
Note that the third identity for $n = -1$, $\nabla(1/r) = -\vec{\hat{r}}/r^2$, describes the electrostatic field and potential of a point charge, the fourth identity is the Gauss law for the electric field, and the fifth identify shows the conservative nature of the electrostatic field.

9) Consider a positive point charge located inside an isolated conducting spherical shell: 1) at the centre of the sphere; 2) at a certain distance from the centre of the sphere. Draw a sketch of the electric field lines in each case.

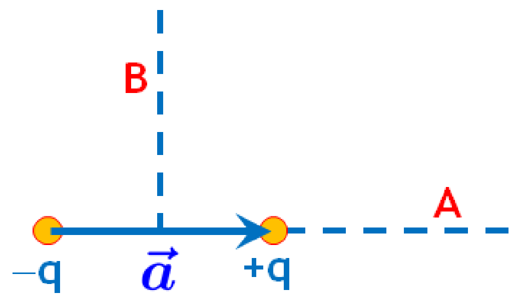
10) A point charge Q is located at a distance d from the centre of a grounded conducting sphere of radius R. Prove that the charges induced on the surface of the sphere are equivalent to a point image charge of magnitude $q = -QR/d$ located inside the sphere, on the line connecting the centre of the sphere and the charge Q, at a distance of $x = R^2/d$ from the centre of the sphere. Find the surface density of the induced electric charge at points A and B on the outer surface of the sphere.



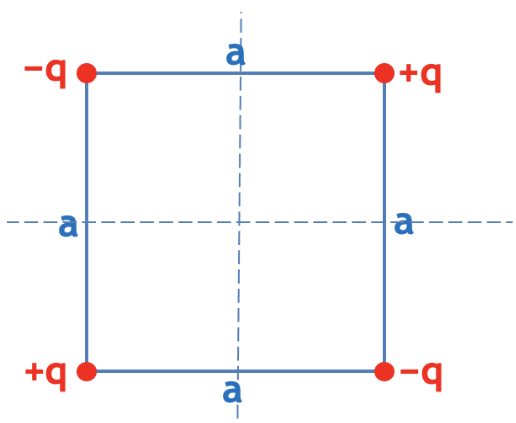
11) It follows from the previous problem that the electrostatic image of a point charge q located inside a grounded conducting spherical shell of inner radius R at a distance x from the centre is a point charge $Q = -qd/R$ located outside the sphere a distance $d = R^2/x$ from the centre. Consider a point-like electric dipole $\vec{p}$ placed at the centre of the spherical shell. Find the direction and magnitude of the electric field at points A and B inside the spherical shell, close to the inner surface.

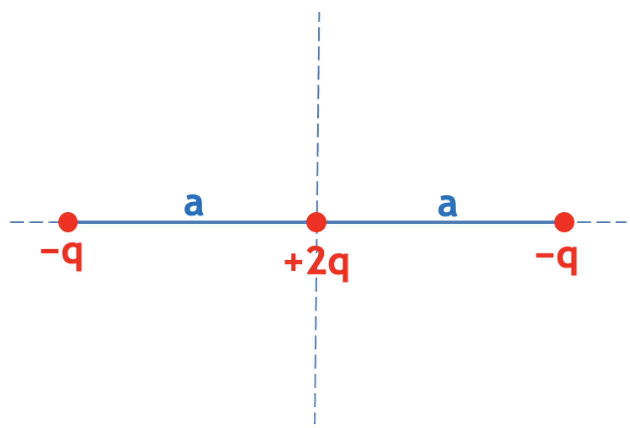


12) Consider a simple electric dipole consisting of two opposite point charges $\pm q$ separated by a distance a. Using Coulomb's law, find the electrostatic potential φ, direction and magnitude of the electric field $\vec{E}$ on the axis of the dipole (A) and in the plane of symmetry perpendicular to the axis (B) at a distance $r \gg a$ from the dipole. Compare the results with those obtained in lecture 4 for the general case. Sketch the field lines.



13) Consider a planar and a linear electric quadrupoles shown below. Using the definition of the dipole moment, check that the dipole moment of each system is zero. Find the direction and magnitude of the electric field on the axes of symmetry shown by the dashed lines, at a distance $r \gg a$ from the centre of the quadrupole.





14) Prove that the force acting on a point-like electric dipole $\vec{p}$ placed into an external electric field $\vec{E}$ can be written as $\vec{F} = (\vec{p}\nabla)\vec{E}$. What happens to a dipole placed into 1) a uniform external field; 2) a non-uniform external field?