Y2 Electromagnetism 2

Non-assessed problem sheet 2 (weeks 3–4)

Magnetostatics – EM induction – Dielectrics

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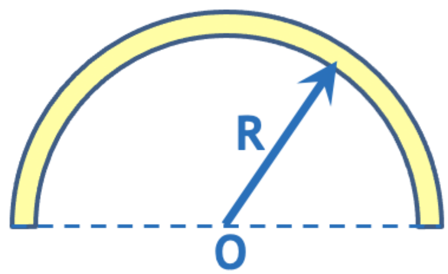
1) Prove that the vector potential for a uniform magnetic field $\vec{B}_0$ is

$$\vec{A}(\vec{r}) = \frac{1}{2}\vec{B}_0 \times \vec{r}.$$

Sketch the field lines of the vector potential $\vec{A}(\vec{r})$.

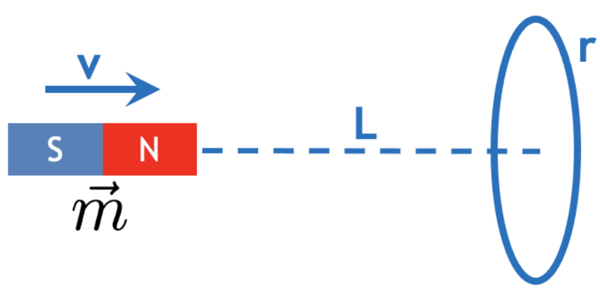
2) Consider a long thin solenoid of radius r, with a surface current density i perpendicular to the axis of the solenoid at each point of the surface. The solenoid is bent to form a semi-toroid of radius $R \gg r$, as shown below. Find the direction and magnitude of the magnetic field at the centre of curvature of the semi-toroid O.

Hint: the bent solenoid can be considered as a system of a large number of thin slices; each slice represents an elementary magnetic dipole.



3) Consider a vertical ($B_x = B_y = 0$), non-uniform magnetic field which depends on the z coordinate only: $\partial B_z/\partial z \neq 0$. A magnetic dipole is placed into the field. The angle between the dipole moment $\vec{\mu}$ and the z axis is θ. Compute the force acting on the dipole. Recall the Stern-Gerlach experiment discussed in the QM2 course.

4) Consider a circular loop of wire of radius r, and a permanent magnet with a dipole moment $\vec{m}$ located on the axis of symmetry of the loop, at a distance $L \gg r$ from the loop. The magnet moves along the axis with a speed v. Find the electromotive force $\mathcal{E}$ in the loop.



5) The atomic polarisability, α, characterises the induced electric dipole moment of an atom placed into an external electric field: $\vec{p} = \alpha\varepsilon_0\vec{E}$ (see lecture 7). Consider a simplified model of the atom: assume that the electron charge is distributed uniformly inside a sphere of radius R, and a point-like nucleus is located at the centre of the sphere. Compute the polarisability α in this model. Using this result, estimate the electric susceptibility of atomic hydrogen at STP. The Bohr radius of hydrogen atom is $R_{\text{Bohr}} = 5.3 \times 10^{-11}$ m.

6) Consider region 1 (defined as $z < 0$) containing a dielectric material with relative permittivity $\varepsilon = 2$, and region 2 (defined as $z > 0$) containing a dielectric material with $\varepsilon = 3$. The electric field in region 1 is uniform:

$$\vec{E}_1 = (-10\vec{e}_x + 20\vec{e}_y + 30\vec{e}_z) \text{ V/m}.$$

Find the electric displacement and polarisation fields $\vec{D}$ and $\vec{P}$ in region 2. Find the surface density of electric charge at the boundary of the two regions.

7) A spherical capacitor consists of a solid spherical conductor of radius $r_1 = 5$ cm, surrounded by a concentric hollow spherical conductor of inner radius $r_3 = 7$ cm. The space within the spherical shell between r_1 and $r_2 = 6$ cm is filled with a dielectric material of relative permittivity $\varepsilon = 7$. The remaining space in the capacitor is filled with air ($\varepsilon = 1$). Compute the capacitance.

8) Find the total energy U of the electrostatic field produced by an electric charge Q distributed on a sphere of radius R in two cases: 1) the charge is distributed uniformly on the surface of the sphere; 2) the charge is distributed uniformly in the volume of the sphere. Assume that $\varepsilon = 1$ at all points in space. Compute the classical electron radius using the first model and assuming that the electron mass is entirely of electrostatic origin: $U = m_e c^2/2$.

9) Using the results from the previous problem, calculate the release of electrostatic energy in the nuclear fission process ${}^{238}_{92}\text{U} \rightarrow {}^{119}_{46}\text{Pa} + {}^{119}_{46}\text{Pa}$. Assume that the electric charge is distributed uniformly in a sphere of radius $R_1 = 10^{-15}$ m in a uranium nucleus, and in a sphere of twice smaller volume, i.e. of radius $R_2 = R_1/\sqrt[3]{2}$, in a palladium nucleus.
