Y2 Electromagnetism 2

Non-assessed problem sheet 4 (weeks 7–8)

EM waves – Poynting vector – Polarisation

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1) Consider a linearly polarised plane monochromatic electromagnetic wave in vacuum:

$$\begin{aligned} E'_x(z,t) &= E_0 e^{i(\omega t - kz)}; \\ B'_y(z,t) &= B_0 e^{i(\omega t - kz)}. \end{aligned}$$

a) Using Maxwell's equations, prove that $B_0 = E_0/c$.

b) Write down the expressions for $E''_x(z,t)$ and $B''_y(z,t)$ for a plane wave of the same amplitude, polarised in the same plane and propagating in the opposite direction.

c) Find the expressions for the sum of the electric and magnetic fields of the two waves. Demonstrate that the resulting wave is a standing wave.

d) Find the amplitude of this standing wave.

e) Establish the main properties of the standing wave, including the location of the nodes of the electric field with respect to the nodes of the magnetic field, and the phase shift between the electric and magnetic fields.

f) Prove that energy does not propagate along the z axis in this standing wave.

2) Using the approach introduced in lecture 14, prove the following identities for the $\vec{E}$ field in a plane monochromatic electromagnetic wave:

$$\nabla \vec{E} = -i\vec{k}\vec{E}; \quad \nabla \times \vec{E} = -i\vec{k} \times \vec{E}; \quad \nabla^2 \vec{E} = -k^2 \vec{E}; \quad \frac{\partial \vec{E}}{\partial t} = i\omega \vec{E}.$$

Note that these identities are also valid for the $\vec{B}$, $\vec{D}$ and $\vec{H}$ fields, in the above conditions.

3) A linearly polarised laser beam propagating in vacuum has a time-averaged energy flux of 50 W/m^2. Compute peak amplitudes of the electric and magnetic fields.

4) Zircon crystals exhibit a high degree of birefringence, with typical refractive indices of 1.96 and 2.01 for the fast and slow axes, respectively. Find the thickness of a zircon plate required to convert left circularly polarised green light (wavelength in vacuum $\lambda = 500$ nm) to right circularly polarised light or vice versa.