Transcript of invited paper presented at the DESCRIPTION AND CREATIVITY Conference, King's College, Cambridge, 4 July 2005

(Supported by Arts Council England, AHRB, and King's College, Cambridge)

THE UNIVERSITY
OF BIRMINGHAM

Goronwy **Tudor Jones**
Centre for Lifelong Learning
The University of Birmingham
Selly Oak
Birmingham B29 6LL
United Kingdom
Tel: 0121 414 4629/5595
gtj@hep.ph.bham.ac.uk

DRAFT 2

4 July 2005

An Introduction to Complementary Descriptions in 20th Century Physics

Abstract

A discussion of what was recently voted 'the most beautiful experiment in physics' - the interference of single electrons in a Thomas Young double slit set-up - will be used to introduce one of the deepest (and most disturbing) realizations of 20th century physics: Niels Bohr's wave-particle complementarity. It will be seen that this has forced physicists to change their ideas about causality and reality.

Introduction

Human creativity tries to come to terms with the mystery of the universe, and our existence within it. Responding to our experiences we have, over the ages, produced creation myths and religious systems, philosophies, art and literature, music, science and technology, medicine, social structures ...

Weisskopf (1989, p.35) points out that these attempts to give meaning to life may seem 'to be incommensurable, mutually exclusive, or even contradictory; I believe, however, a better word is *complementary* ... they represent different aspects of reality, one aspect excluding the other, yet each adding to our understanding of the phenomenon as a whole.'

The aim of this talk is to introduce, to the non-scientist, one of the most fundamental discoveries of 20th century physics - the Principle of Complementarity. This manifests itself in many ways; here we limit ourselves to one aspect, wave-particle complementarity, because this can be presented in terms of a simple experiment - Young's 2-slit experiment with electrons.

This was created by the Nobel Laureate Richard Feynman (1964, ch.37) as a thought experiment to describe the mysterious behaviour of subatomic particles: indeed, Feynman went so far as to claim that 'In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics'. Since then, the experiment has actually been performed (Jönsson, 1974); it gave the results described by Feynman, and was recently voted 'the most beautiful experiment in physics' (Crease, 2002) We will see how physicists, by accepting the authority of experiment and giving up their cherished principle of ('strict') causality[1], one of the ideas that founded the Enlightenment, made what they regard as the leap that has most extended mankind's understanding of the workings of nature.

---

[1] **'Strict' Causality;** if at some instant we know (i) the position of an object, and (ii) its speed and direction of motion, then, using Newton's Second 'Law' which tells us how the motion of an object is changed by a force, we can predict its subsequent motion, moment by moment.

The Two-slit Experiment with Bullets

Imagine some idealised, indestructible bullets being fired horizontally at a wall through a bullet-proof screen with two horizontal rectangular slits in it. Imagine further that all this takes place in a perfect vacuum, so that there is no air resistance. Then, following Galileo and Newton, if the bullets are moving so quickly that their fall under gravity can be neglected, we would expect the bullets to end up in two horizontal rectangular piles in front of the walls and behind the slits. See Fig. 1, in which the piles are represented by a graph showing the number arriving as a function of position on the wall.

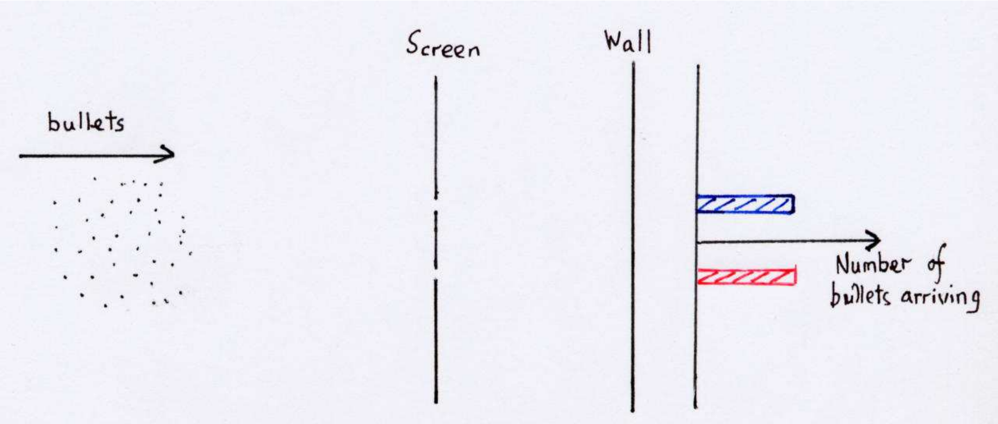


Fig. 1

We will follow Feynman and present in Fig. 2 a slightly more realistic version which allows for a slight ‘spraying’ effect; it shows two bumps representing separately the expected distribution of arrivals of bullets that have come through the upper (blue) and lower (red) slits.

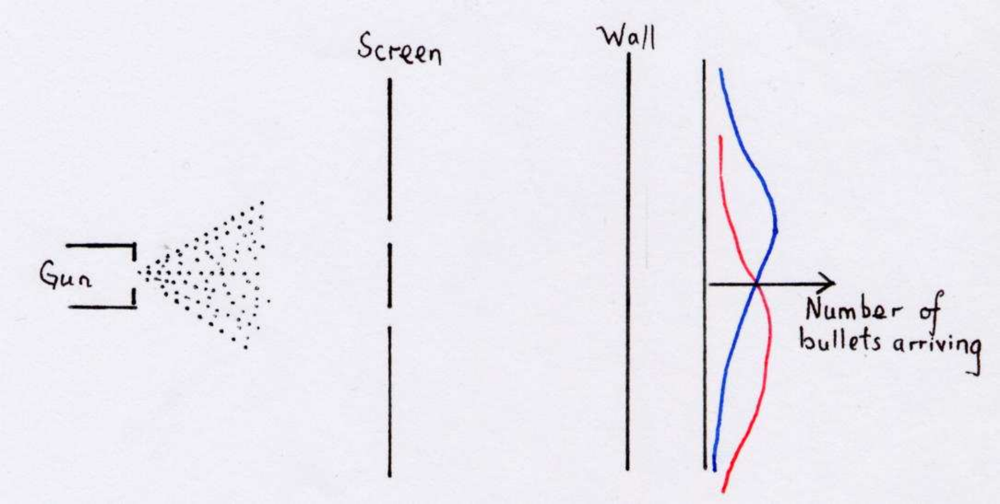


Fig. 2

In Fig. 3 the green line is a straightforward addition of the individual arrival bumps and represents the overall distribution of bullet arrivals.

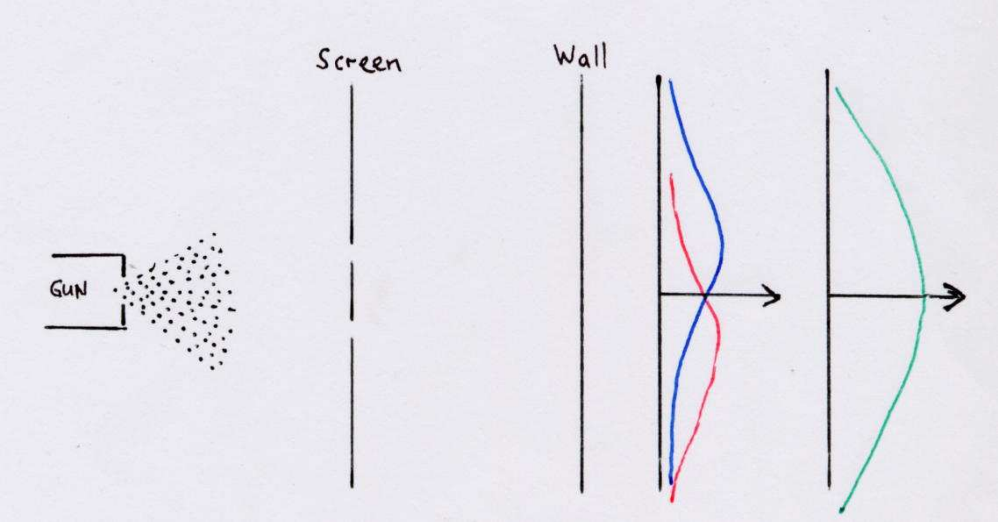


Fig. 3

Two-slit Experiment with Electrons

If we perform a similar experiment with a beam of the tiniest particles known - electrons[2] - shot through slits much thinner than a hair's breadth[3] the result is utterly different: instead of a smeared-out distribution of arrivals like the green line of Fig. 3, which is what we would expect if Newtonian mechanics described how electrons move, we get (see Fig. 4) alternate regions of low and high arrival rates: the region straight ahead of the gun gets most electrons; as we move away, either up or down, the signal (number of electrons arriving) decreases, increases, decreases ...

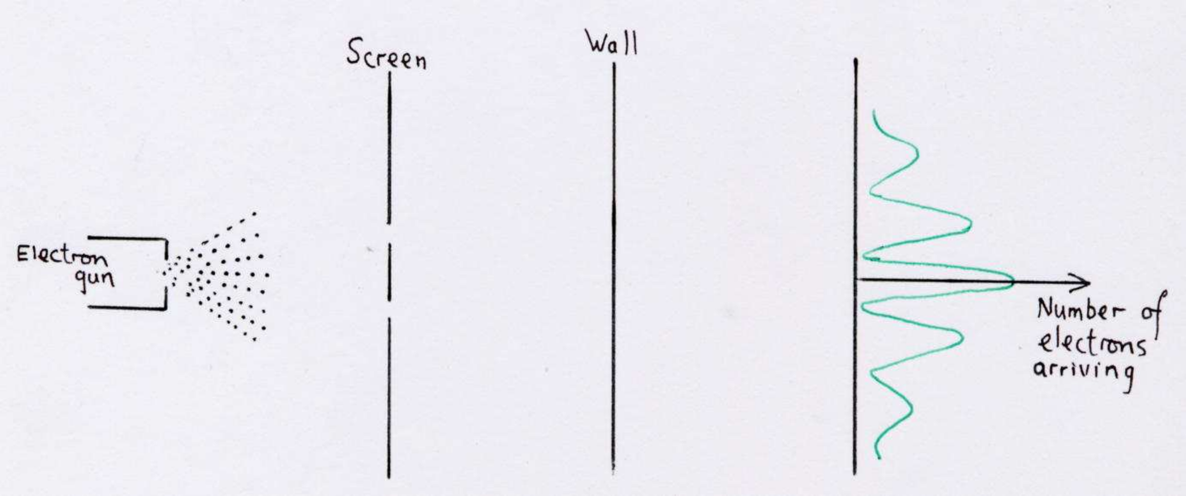


Fig. 4

---

[2] A television shoots beams of electrons at the screen to create the picture, using a tube which is similar to the apparatus used by J J Thompson when he discovered the electron in 1897.

[3] The width of the slits used were about 300 times narrower than a hair's breadth, separated by a distance of about $\frac{1}{100}$ of a hair's breadth.

Interference of waves

To make progress with this bizarre experimental discovery we need to learn something about the nature of waves. We are all familiar with the beautiful patterns we can get by dropping pebbles into still water - these ‘interference patterns’ are formed when the circular waves caused by the individual pebbles run through each other.

Similar patterns can be formed when waves are passed through narrow gaps. Fig. 5 is a diagram representing what happens when a regular sea wave - which we are viewing from above as it moves from left to right - encounters a harbour wall with two gaps in it; the parallel red lines represent successive crests.

To the right of the harbour wall with two gaps in it, we see the interference pattern of the two sets of circular waves that start from the gaps. The water surface is illuminated from above and to the left; so the alternate light and dark regions correspond to the sides of the waves that are illuminated and in shade respectively. See, for example, the directions labelled wave. Let us describe the pattern. As we move away from the wave moving in the same direction as the original wave, we see alternate regions of waves and no waves (grey) - strong signal, weak signal, strong, weak ...

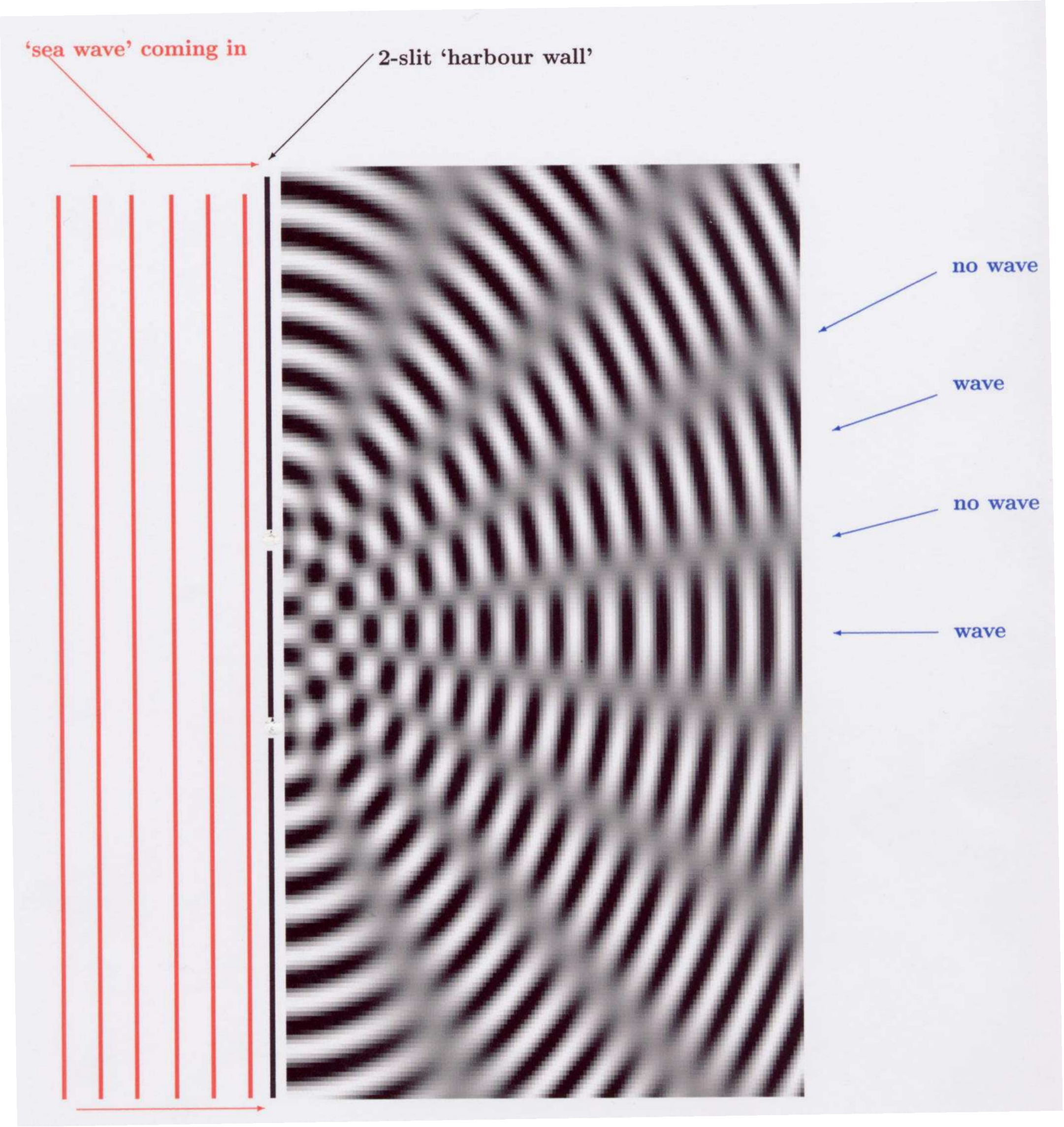


Fig. 5

This is reminiscent of the distribution of electron arrivals in the 2-slit experiment - the green line of Fig. 4. Let us in Fig. 6, which has circular rather than straight waves because the source is close to the screen, re-cast Fig. 5 in a more familiar way:

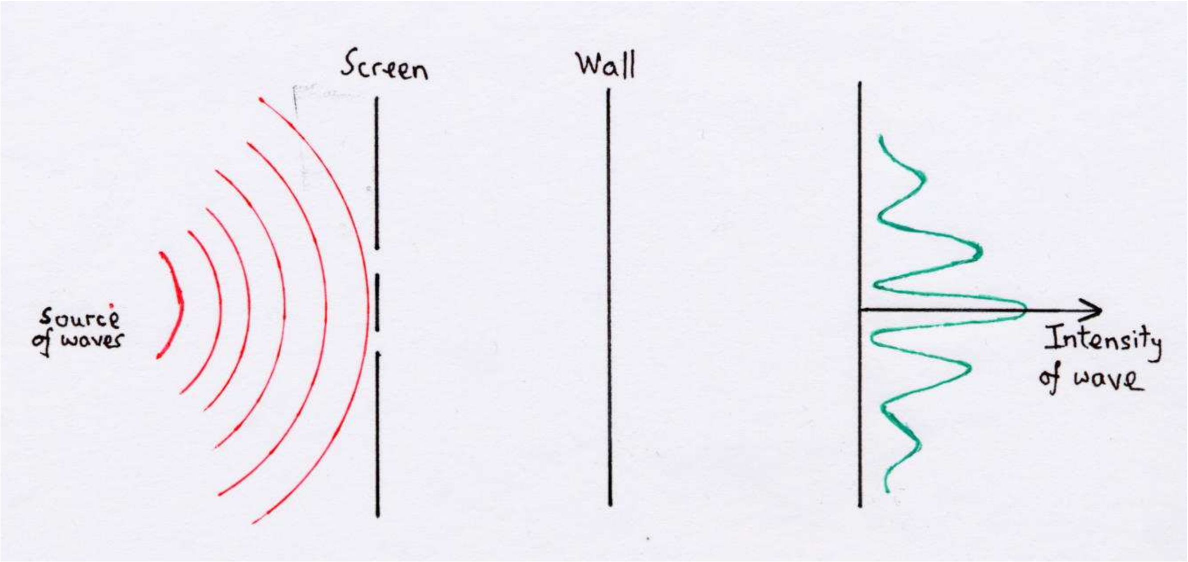


Fig. 6

If we pass red light from a laser through two slits, separated by a few tenths of a millimetre, the intensity of the light arriving at the wall, as seen from behind the laser, is as shown in Fig. 7:

Again we see a strong signal in the direction of the original beam (centre of picture), which, as we move away, weakens to a minimum, grows to maximum, weakens, grows ...

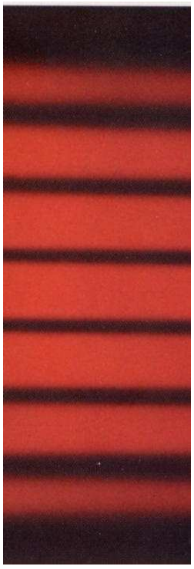


Fig. 7

It was in 1803 that Thomas Young, who was familiar with the way water waves form patterns, performed the first ever 'two-slit' experiment to demonstrate that light was a wave - not 'corpuscles', or particles as we would say now, as Newton has posited. Young used white light (sunlight) and, as a result of the fact that the different components of light have different wavelengths, the minima (dark lines in Fig. 7) would not have coincided; this would then have resulted in a multicoloured smear.

Back to the Two-slit experiment with Electrons. Complementarity.

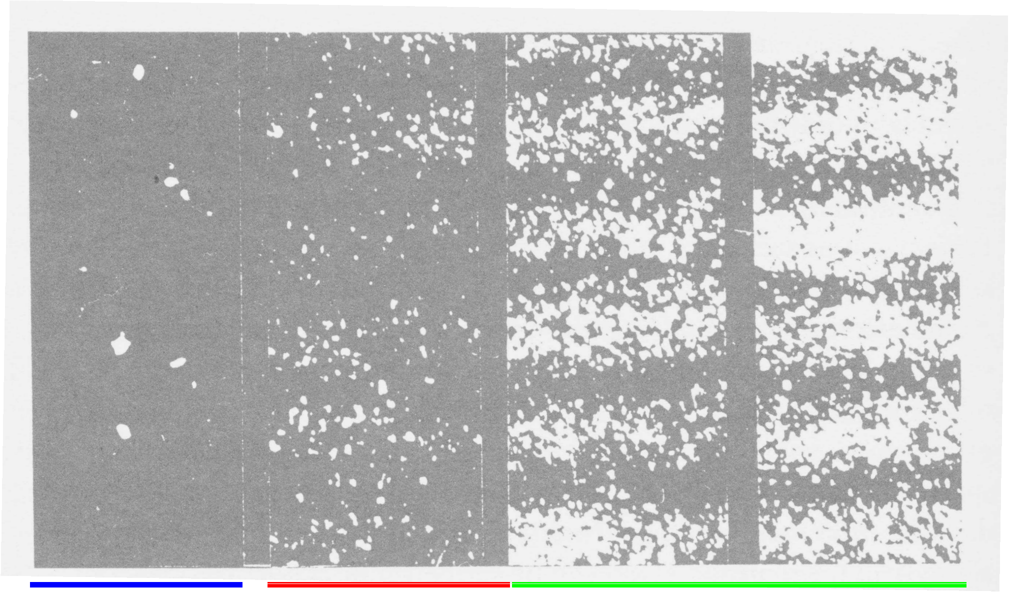
Let us now return to the result of performing the 2-slit experiment with electrons, the green line of Fig. 4.

It tells us that:

- The distribution of electron arrival points is not what one would expect from treating them as **particles** to which we could apply Newton's 'laws' (which would give the green line of Fig. 3).
- The distribution of electron arrival points[4] matches the distribution of intensity of **waves** that have passed through a 2-slit arrangement.

NB The experiment has been performed in such a way that only one electron passes through the apparatus at any one time. The result represented by Fig. 4 is the same, even though each electron is recorded as an individual, localised particle.

The experimental data from [3] is shown in the photograph shown in Fig. 8:



Longer exposures - showing pattern ever more clearly.

$\sim$10 seconds - begin to see suggestion of the high-count-then-low-count pattern.

$\sim\frac{1}{4}$ second - sparsely-populated pattern of spots corresponding to electron hits; looks random.

Fig. 8

---

[4] The 'wall' is coated with a chemical that emits a flash of light at the point where it is struck by an electron.

These four images show the detection of electrons that have passed through the two closely-spaced slits:

- the first, lasting only about a quarter of a second, leaves a sparsely-populated pattern of spots corresponding to electron hits, which looks quite random;
- in the second exposure, lasting about 10 seconds, we begin to see a suggestion of the high-count-then-low-count pattern;
- the 3rd and 4th images, corresponding to longer exposure, show the pattern ever more clearly.

One could conclude then, that although we detect the electrons one-by-one as localised particles, the fact that we have a characteristic 2-slit 'interference pattern' of arrival destinations on the wall shows that, in some sense, each electron must have passed through both slits!

A natural question to ask at this point is whether it would be possible to do an experiment to detect which slit the electron had gone through. Let us follow Feynman's argument in [2].

Electric charges (electrons are negative) scatter light. So, if we put a light source near the slits as shown in yellow in Fig. 9, we should see a flash from near one slit or the other if an electron goes through.

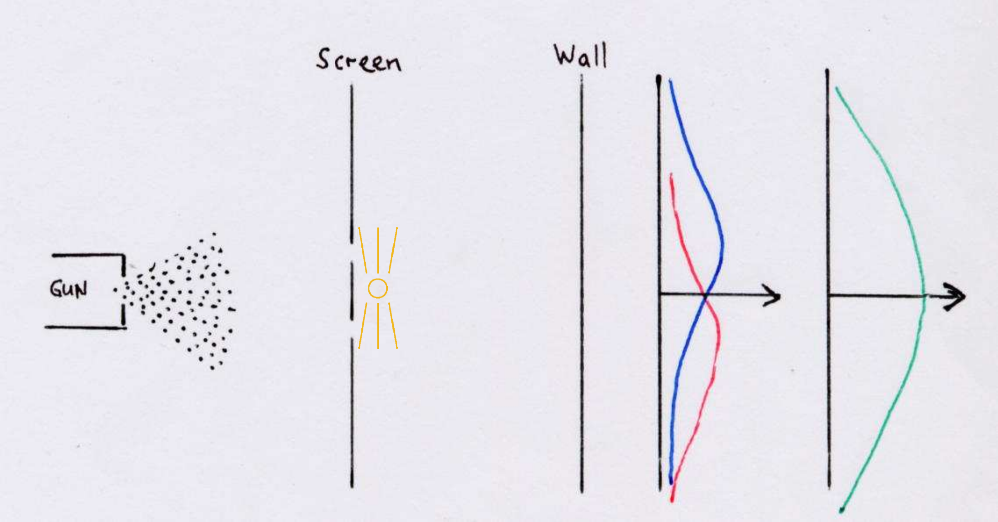


Fig. 9

What we find is the following:

- Every time we detect an electron at the wall, we also see a flash of light either near slit 1 or near slit 2, but never both.

  We conclude: when we perform an experiment to look for electrons, we find that the electron goes through one slit or the other.

- If we keep track of where these electrons end up, we can plot a graph of the the electron arrival points on the wall via each slit individually - upper blue graph for electrons that went through the upper slit, lower red graph for those that went through the lower slit. See Fig. 9.
- By adding these two graphs we can get the green graph of Fig. 9 which represents the arrival point distribution for electrons that arrive via either slit.

- Note: this does not have the 'interference pattern' that we had before, when we did not look. If we switch off the lamp and stop looking to see which hole the electron goes through, the interference pattern comes back!

  What is happening is that when the light hits the electron, it disturbs it enough to destroy the interference pattern - and no-one has found a way round this. It is a fundamental property of nature.

This is telling us that, on the atomic scale, what we find out about nature depends on what question we ask! If we perform one experiment we find that the electron 'behaves' like a particle; perform another and the electron 'behaves' like a wave.

These are two mutually exclusive aspects of the nature of the electron which, taken together, 'add to our understanding of the phenomenon as a whole'.

This experimental finding is called **wave-particle complementarity** and applies to all particles, not just electrons.

> *We cannot at the same time experience the artistic content of a Beethoven sonata and also worry about the neurophysiological processes in our brains.*
>
> Victor Weisskopf

Complementarity and Complementarism

The success of quantum mechanics, with its staggeringly detailed insights into the workings of both the subatomic and cosmological worlds (we see the universe through the eye of the atom), insights that also spawned our high-tech industries, has convinced physicists that its foundation, complementarity, is here to stay.

Throughout his life, Bohr felt that complementarity might bear fruit if applied outside physics - he often used the phrase 'the epistemological lesson of quantum theory'. This is discussed at some length in an excellent book (Pais, 1991). Bohr regarded his extrapolations into other fields - dubbed 'complementarism' by Pais - as 'seeds for further thought' (Pais 1991, p.439).

Finally, with a few examples from Pais (1991, pp. 439-447), an attempt is made to give a sense of how 20^{th} physics is in tune with the idea that the diversity of human experiences is best addressed by a complementary approach, in which different aspects of a subject are brought together, each one 'excluding the other yet adding to our understanding of the whole'. (Weisskopf, 1989, p.62)

Complementary aspects of psychology

> The fact that consciousness, as we know it, is inseparably connected with life ought to prepare us for finding that the very problem of the distinction between the living and the dead escapes comprehension in the ordinary sense of the word. That a physicist touches upon such questions may be perhaps excused on the ground that the new situation in physics has so forcibly reminded us of the old truth that we are spectators as well as actors in the great drama of existence.
>
> Niels Bohr, 1929

Pais comments on this summary of Bohr's ideas on complementary aspects of psychology:

> A person contemplates, is spectator, when planning his action, and again when reflecting on its results. In between, when acting, he is, one hopes, also thinking but not in the contemplative mode. To be spectator is as necessary for executing and evaluating the role of actor as to perform the act itself. These two modes of engagement are both necessary elements in the person's mental content, yet the exclude each other - they are complementary.

Another aspect of Bohr's thinking about about psychology involves his regarding emotion and reason as complementary. He, for example, finds a place for *free will* on the emotional/feeling side.

Complementarity and Communication/Language

Communication of ideas was important to Bohr, but a great communicator he was not. Pauli has written of him:

> He knew what he wished *not* to say when he strove in long sentences to express himself in his scientific papers.

It may be said, I think, that what stood in Bohr's way of expression was complementarity itself. How can a man talk and write in simple terms when he believes that 'the conscious analysis of a concept stands in an exclusive relation to its immediate application'? Or that 'the practical use of every word stands in a complementary relation to its strict definition'?

As Rosenfeld has recalled: '[Bohr] felt that whenever you come with a definite statement about anything you are betraying complementarity ...'.

Although the examples of complementarism mentioned seem to mirror the complementarity of physics by appearing in pairs, the main message conveyed is that there are different ways of examining something, each giving different insights.

Reflections on Creativity

When he was Director General of CERN, Weisskopf gave a public lecture on the work of the laboratory, during the course of which he gave a succinct definition of science - he was thinking particularly of physics.

The gist of it was that science functions by

- DISCOVERING - by means of experiments and observations;
- DESCRIBING - by means of models or theories (usually expressed in the language of mathematics);
- APPLYING the insights gained by the above processes to develop tools with which to further both science and society.

Following Feynman, we have described the mysteries of the sub-atomic world in terms of the 2-slit experiment with electrons. Some would argue that it was very creative to try to extract the essence of an essentially mathematical theory - quantum mechanics - in a visualisable way. Having decided to take this route, it seems reasonable to say that the description of the experiment itself is not particularly creative.

However, if we start with the experiment we can measure a few things such as:

- the angle to the first strong signal from the central one
- the detailed features of the signal we get when only one slit is open (the Heisenberg Uncertainty Principle is hidden here).

These measurements can be summarised in a few simple mathematical relationships. If we then make the speculative assumption that the waves of wave-particle duality obey the same mathematics (have the same properties) as all other waves, we can incorporate our relationships into the body of classical (19th century) physics and see what happens.

The two images in Appendix A show how without invoking further physics we can estimate the size of the hydrogen atom (an electron confined to the vicinity of a proton) using high-school mathematics. One can go further, and estimate the amount of energy needed to take an electron out of a hydrogen atom. Both estimates agree with experiment.

Appendix A is a poor-man's Schrödinger Wave Equation calculation. Using the same physics but a more powerful mathematical description, Schrödinger was able to show how electrons in atoms can only have certain energies (unlike satellites that can be put into orbits at any height). This realisation, in turn, houses what is possibly the most amazing insight of quantum physics: when atoms are smashed, as is happening in our bodies all the time as cosmic rays force their way through us, the bits - electrons, protons, ions - come together again to produce atoms that are indistinguishable from the original ones; they can only be that way! Without this, we could not exist.

Here we see that the mathematical description is creative - it gives insights into the nature of the subatomic world that could not have been foreseen. As Paul Dirac said in another context (when his relativistic generalization of the Schrödinger equation predicted the existence of anti-matter which we now use routinely in PET[5] scanners in our hospitals), the equations seem 'more intelligent than their author'.

The power of mathematics to describe the behaviour of the material world in intricate detail, and to predict the existence of hitherto unimagined phenomena, is one of the great mysteries of life. (Wigner, 1960)

---

[5] **Positron Emission Tomography (PET)** The use of the annihilation of positrons (anti-electrons) by electrons in a PET scanner is a wonderful example of the creativity of the applied scientist. We have new materials; let's see what we can make with them: a non-invasive tool that can pin-point cancer cells to within millimetres using anti-matter engineering.

References

1. CREASE, R.P., 2002. The double-slit experiment. *Physics World*, Sepember 2002, 15-17.

2. FEYNMAN, R.P., LEIGHTON, R.B., and SANDS, M., 1964. *The Feynman Lectures on Physics Volume I.* Reading Massachusetts, Palo Alto, New York:Addison-Wesley.

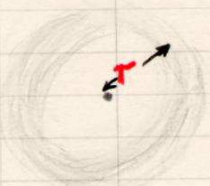
3. JÖNSSON, C.J., 1974. Electron Diffraction at Multiple Slits. *American Journal of Physics*, 42, 4-11.

4. PAIS, A., 1991. *Niels Bohr's Times, In Physics, Philosophy, and Polity.* Oxford: Clarendon Press.

5. WEISSKOPF, V.K., 1989. *The Privilege of Being a Physicist.* New York: W.H. Freeman.

6. WIGNER, E., 1960. The Unreasonable Effectiveness of Mathematics in the Natural Sciences. *Communications in Pure and Applied Mathematics*, 13, No. 1.

The Hydrogen Atom



The energy of an electron confined to the vicinity of a proton by electrical attraction is given by

$$E = \text{kinetic energy} + \text{potential energy}$$

$$= \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} \qquad \mathrm{I}$$

The position of the electron is 'uncertain' by an amount that is roughly $\frac{r}{2}$, half the 'radius' of the atom; i.e. $\Delta r \sim \frac{r}{2}$

By the Heisenberg Uncertainty Principle, the corresponding uncertainty in momentum is

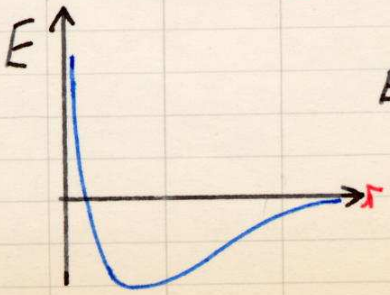
$$\Delta p \sim \frac{h}{2\pi r}$$

But the average momentum p of the electron must be zero (otherwise the electron would leave the proton!); so

$$p \sim \frac{h}{2\pi r} \qquad \mathrm{II}$$

Substitute II into I to get

$$E = \frac{h^2}{8\pi^2 m r^2} - \frac{e^2}{4\pi\epsilon_0 r}$$



The favoured value of r is the one that minimizes the energy, which we get by putting $\frac{dE}{dr} = 0$

ie
$$\frac{-2h^2}{8\pi^2 m}\cdot\frac{1}{r^3} + \frac{e^2}{4\pi\epsilon_0 r} = 0$$

ie
$$\underline{r(\text{minimum energy}) = \frac{h^2\epsilon_0}{\pi e^2 m}}$$

Put in the numbers:

$h = 6.63 \times 10^{-34}$ Js
$\epsilon_0 = 8.85 \times 10^{-12}$ F/m
$e = 1.6 \times 10^{-19}$ C
$m = 9.11 \times 10^{-31}$ kg

$$\underline{\underline{r(\text{minimum energy}) = 0.53 \times 10^{-10}\ \text{metre}}} \qquad \text{III}$$

Comparable to what we get from the spreading oil drop experiment!

- To get the minimum energy put III into I

to get
$$E = \frac{h^2}{8\pi^2 m}\,\frac{\pi^2 e^4 m^2}{h^4\epsilon_0^2} - \frac{e^2}{4\pi\epsilon_0}\,\frac{\pi e^2 m}{h^2\epsilon_0}$$

$$= -\frac{1}{8}\,\frac{me^4}{h^2\epsilon_0^2}$$

Substituting the numbers we get

$$\underline{\underline{E = 13.6\,\text{eV}}}$$

EXACTLY THE ENERGY NEEDED TO GET AN ELECTRON OUT OF A HYDROGEN ATOM!