Some suggested Projects

1. Random numbers are very useful for 'Monte Carlo' simulations of physical problems. Some possibilities are:

a. Write a program to simulate the National Lottery.

b. Simulate radioactive decays. (Choose a random number z between 0 and 1, as in ran.cpp. A decay occurs in a short time interval Δt if $z \le \lambda \Delta t$., where λ is the decay constant Can you analyse the statistical distribution of decays and the time-interval between decays?)

c. Use the Gaussian random-number generator to simulate the Brownian motion of a particle. (For a free particle, the displacement x in one dimension obeys an equation of the form $x(t+\Delta t) = x(t) + \xi\sqrt{2D\Delta t}$, where ξ is a Gaussian random number of unit variance and D is the diffusion coefficient. How far does the particle diffuse in time t?)

2. Other problems can be simulated by numerical integration of differential equations. The Runge-Kutta algorithm 'rkint' in rkdemo.cpp works well for many simple cases of coupled first-order differential equations:

a. Write a program to simulate a non-linear oscillator described by an equation of the form

$$d^2x/dt^2 - d/dt(\alpha x - \beta x^3) + \omega_0^2 x = 0,$$

where α, β and ω_0 are constants. (This is the van der Pol equation - see, for instance, chapter 4 of Laser Physics (1974) by Sargent, Scully and Lamb. Look at the behaviour when $\omega_0 >> \alpha$ and $\omega_0 << \alpha$.)

b. A classic Physics problem is to analyse a system of identical coupled pendulums (each one joined to its neighbour by a spring). Develop a program that can handle from two pendulums upwards, and look at what happens when you give a pendulum at one end a kick.

c. Develop a simulation for another coupled system such as a three- or four-level laser, or radioactive series.

3. a. Use a fast Fourier transform to analyse the Gaussian noise in 1c. Bear in mind that you will need to average the spectral density for many samples to get an accurate noise spectrum.

b. Modify the FFT program to produce a two-dimensional Fourier transform. This can be used to model Fraunhofer diffraction (e.g., by a circular aperture) or a two-dimensional hologram.

4. Develop a C++ matrix class that can handle basic matrix operations.