Lecture 3: Propagators

0 Introduction to current particle physics

1 The Yukawa potential and transition amplitudes

2 Scattering processes and phase space

3 Feynman diagrams and QED

4 The weak interaction and the CKM matrix

5 CP violation and the B-factories

6 Neutrino masses and oscillations

7 Quantum chromodynamics (QCD)

8 Deep inelastic scattering, structure functions and scaling violations

9 Electroweak unification: the Standard Model and the W and Z boson

10 Electroweak symmetry breaking in the SM, Higgs boson

11 LHC experiments

Recap of Lecture 2 :

A massive spinless particle being exchanged as part of a force field can be described by a quantum mechanical wavefunction that dies faster than 1/r

Yukawa potential
(spherical symmetric static case)

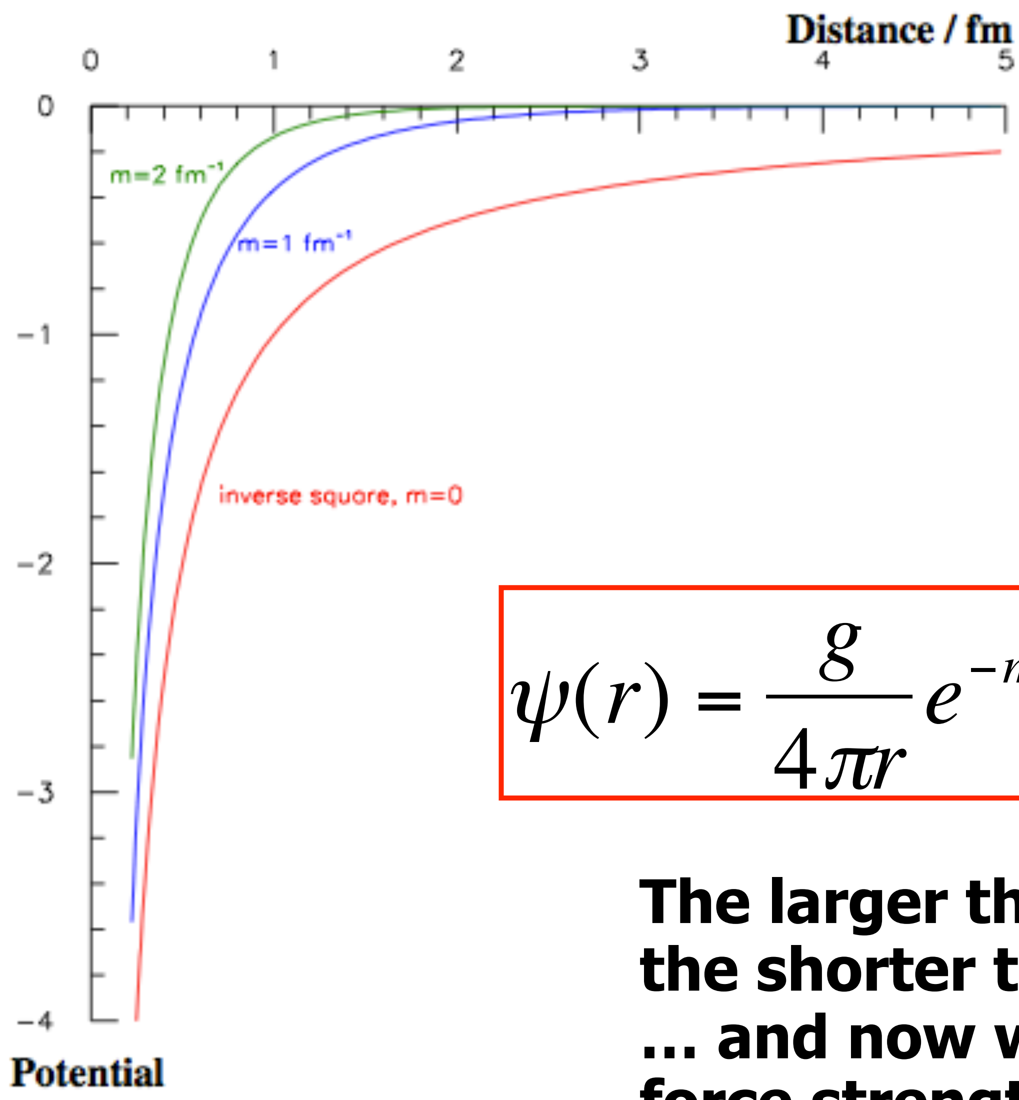
$$\psi(r) = \frac{g_0}{4\pi r} e^{-mr}$$

m=1/R = mass of particle carrying the force
R = range
g_0 = fundamental strength (coupling) of the force

Mass and range are related: the larger the mass exchanged, the shorter the range … and if m = 0, R = ∞ (though with $1/r^2$ decay of the force strength)

Today: calculate scattering amplitude for particles incident on a potential, taking Yukawa potential as an example

(From the end of Lecture 2 …)
Yukawa Potential



$$\psi(r) = \frac{g}{4\pi r} e^{-mr}$$

In case of photon, electric potential, m=0 and R=∞

For m>0, force has range R: beyond R, V tends to zero quickly because of exponential factor

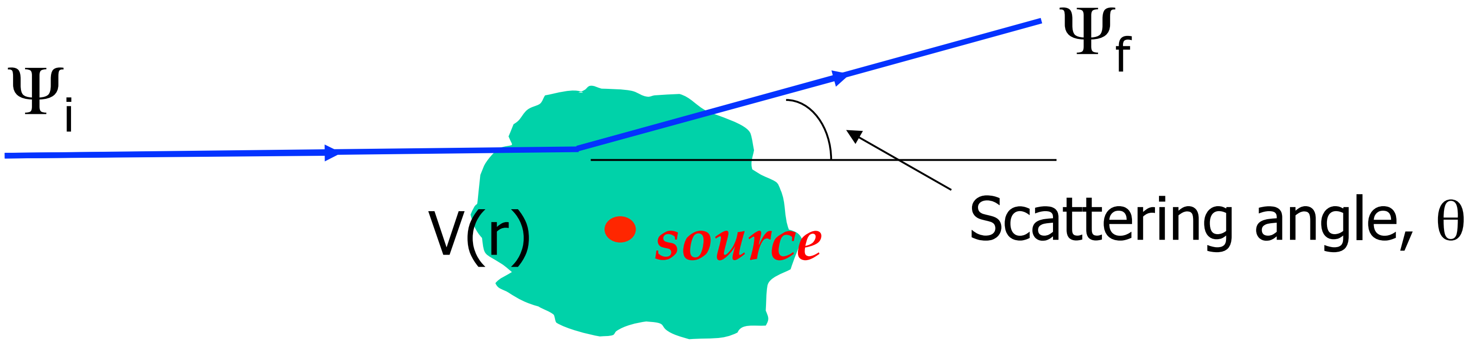
The larger the mass exchanged, the shorter the range …
… and now we have a model for force strength versus distance

Scattering of Plane Waves

Much of particle physics is about calculating how likely things are to happen (decay probabilities, reaction cross sections …)

These correspond to transitions between quantum mechanical states

Consider transition of an incoming particle i to a final state particle f due to the presence of a force field U(r), which we've identified with the Yukawa potential



Incident and scattered particles can be described by plane waves
→ same solution for Schrodinger and Klein-Gordon …

$$\psi = \psi_0 e^{i(\mathbf{p}.\mathbf{x} - Et)} = \psi_0 e^{-ip_\mu x^\mu}$$

Scattering: Some assumptions / approximations

- Target particles separated by distances >> De Broglie wavelength of incident particles ⇨ no interference

- Target low density ⇨ no multiple scattering

- Collision energy high ⇨ binding energy in target neglected

- Beam intensity low ⇨ mutual interactions in beam neglected

- Force is weak enough that single-exchange dominates:

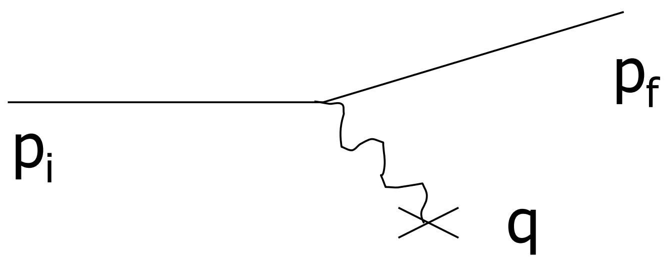
Transition Amplitudes

- Probability of transition from i to f for a single particle passing potential V(r) is described by a transition amplitude (aka matrix element), M_{fi}
- In Born approximation / 1st order perturbation theory:

$$M_{fi} = \int \psi_f^* V(r) \psi_i \, d^4x = \langle \psi_f | V(r) | \psi_i \rangle$$

- For plane waves in and out

$$M_{fi} = \int e^{ip_{f,\mu} x^\mu} V(r) e^{-ip_{i,\mu} x^\mu} d^4x = \int e^{-iq_\mu x^\mu} V(r) d^4x$$



NB: M_{fi} is (the 4D generalisation of) the Fourier transform of the target potential, V(r) … just as in optical diffraction patterns.

$$q_\mu = p_{i,\mu} - p_{f,\mu}$$ q = 4-momentum transfer

Solution for the Scattering Amplitude

Substituting Yukawa potential:

$$\psi(r) = \frac{g_0}{4\pi r} e^{-mr}$$

$$V(r) = g\psi(r) = \frac{gg_0}{4\pi r} e^{-mr}$$

… where g is the fundamental strength of coupling of the incoming particle to the force field.

$$M_{fi} = \frac{gg_0}{4\pi} \int e^{-iq_\mu x^\mu} \frac{1}{r} e^{-mr} d^4x$$

See handout for how to do this integral (non-Examinable)

$$M_{fi} = \frac{gg_0}{q_\mu q^\mu - m^2}$$

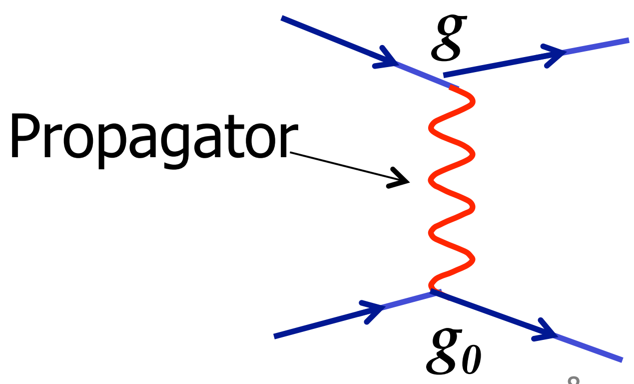
The Propagator

$$M_{fi} = \frac{g g_0}{q^2 - m^2}$$

- g and g0 are the coupling constants describing the fundamental strength of the interaction → not known a priori

- $1/(q^2+m^2)$ is known as the `propagator' and describes the force in terms of the mass of the exchange particle and the squared 4-momentum transfer, as described by the Yukawa potential

- This is the entire basis of Feynman diagrams, as we will see in much more detail in the following lectures



- Caveat: we started from the Klein-Gordon equation, so we're still neglecting spin

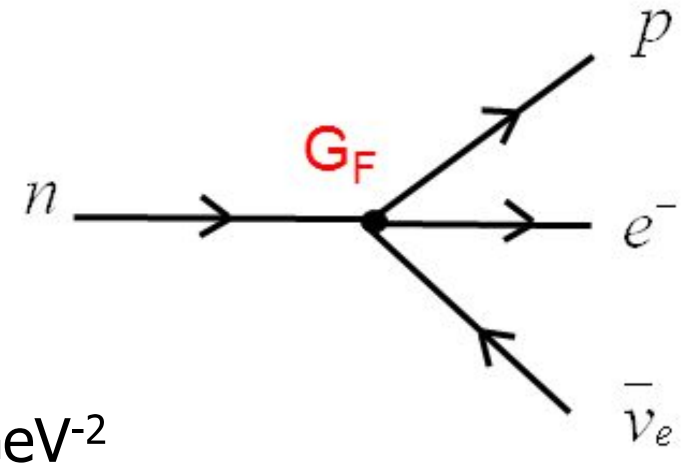
A Special Case – Zero Range Approximation

Propagator … $$\frac{1}{q^2 - m^2}$$

In the `zero-range approximation', $m^2 >> q^2$ and the propagator reduces to a constant $1/m^2$

We then have a point-like `contact interaction' with a single dimensioned coupling, which appears to be a constant

Classic example is the weak interaction in nuclear β decay, where q^2 is very small compared with M_W^2



Fermi constant $G_F = g^2/M_W^2 = 1.166 \times 10^{-5}$ GeV^{-2}

Summary

We have seen how to calculate a matrix element which consists of a propagator times the two "charges" of the scattering particles :

$$M_{fi} = \frac{g g_0}{q^2 - m^2}$$

The ingredients are now in hand to calculate the quantities we measure in particle physics: cross-sections, angular distributions, decay rates etc.

… but first we need to convert transition amplitudes into measurable quantities such as event rates and cross sections

A Note on Signs

- Every text book seems to have a different convention on the sign of $q_\mu = \pm(p_{i,\mu} - p_{f,\mu})$ and on the relative sign of energy and Momentum terms in the metric tensor (+---) or (-+++)

- There is no absolute answer to these questions – only squares of these
- choices are ever physically observable.

The handout on `Fourier transform of Yukawa potential' is a 3D calculation … q^2 there is the 3-momentum squared. Then the propagator is correctly written as $1/(q^2+m^2)$

When we go to 4 dimensions, with our convention of $q^2=E^2-\mathbf{p}^2$, the relative minus sign leads to a change of sign.

The propagator is therefore correctly stated as … $\dfrac{1}{q^2 - m^2}$

Note that for a "space-like" or "exchange" particle, as we have here: $q^2<0$ … so $q^2-m^2<0$ … but propagators always get squared in calculating observables such as cross sections, which are positive-definite