Lecture 6:Feynman diagrams and QED

0 Introduction to current particle physics

1 The Yukawa potential and transition amplitudes

2 Scattering processes and phase space

3 Feynman diagrams and QED

4 The weak interaction and the CKM matrix

5 CP violation and the B-factories

6 Neutrino masses and oscillations

7 Quantum chromodynamics (QCD)

8 Deep inelastic scattering, structure functions and scaling violations

9 Electroweak unification: the Standard Model and the W and Z boson

10 Electroweak symmetry breaking in the SM, Higgs boson

11 LHC experiments

Recap

We have been working towards a systematic method of calculating cross sections such as AB→CD mediated by a given exchange particle.

We calculated a transition amplitude or 'matrix element' which consists of a propagator describing the exchange and the two couplings of the exchnge to the scattering particles (i.e. their `charges'). For a spin-less particle :

$$M_{fi} = \frac{g g_0}{q^2 - m^2}$$

Fermi's Golden Rule allows us to convert this to an event rate prediction, given a knowledge of Lorentz Invariant Phase Space

$$\Gamma_{fi} = 2\pi |M_{fi}|^2 \rho_f(E_i)$$

Finishing off from last time (→ cross sections)

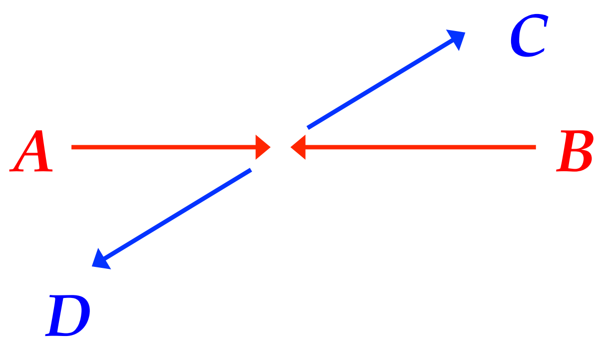
$$\mathrm{d}\sigma = \frac{\mathrm{d}\Gamma_{fi}}{(v_A + v_B)} = \frac{2\pi}{(v_A + v_B)} \left|M_{fi}\right|^2 \frac{2E_C 2E_D}{(2\pi)^6} \frac{\mathrm{d}^3 p_C}{2E_C} \frac{\mathrm{d}^3 p_D}{2E_D}$$

Finally, work in centre of mass frame, such that …

$$s = (E_A + E_B)^2$$

$$|p_A| = |p_B| = p_i$$

$$|p_C| = |p_D| = p_f$$



- Kinematics … $v_A + v_B = \frac{p_i \sqrt{s}}{E_A E_B}$

- Converting from d^3p_c d^3p_D to dΩ (Thomson 3.4.1, 3.4.2 or H&M 4.3) …

$$2E_A 2E_B 2E_C 2E_D \frac{\mathrm{d}^3 p_C}{2E_C} \frac{\mathrm{d}^3 p_D}{2E_D} = (2\pi)^3 \frac{p_f}{4\sqrt{s}} \mathrm{d}\Omega$$

… where dΩ is the element of solid angle around e.g. p_C

Putting ingredients together

$$\frac{d\sigma}{d\Omega}_{cm} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|M_{fi}|^2$$

… provides a method of comparing a theoretical calculation (of $|M_{fi}|^2$) with an experimental measurement (of $d\sigma/d\Omega$).

1/s dependence from kinematics … (accelerators need to increase luminosity to compensate for cross sections falling)

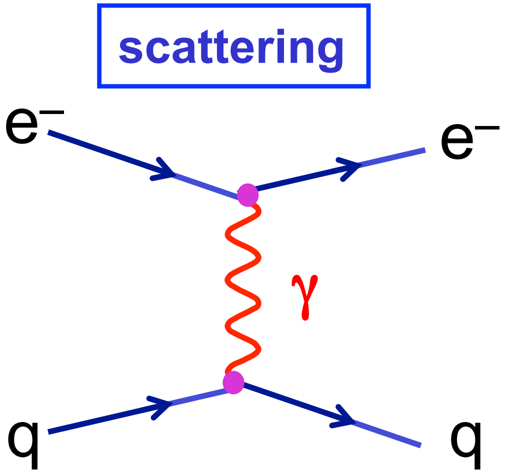
M_{fi} contains all the physics dynamics … couplings and propagators

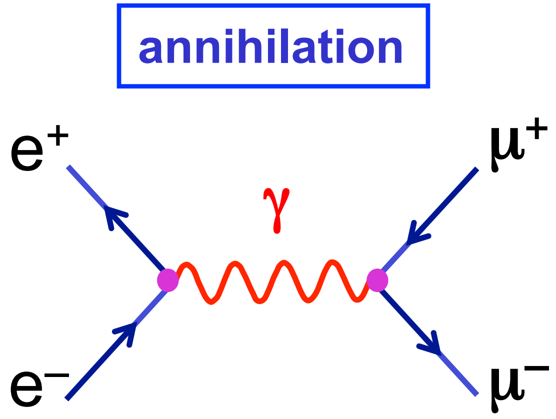
Note that in the case of elastic scattering (AB=CD) or in the ultra-relativistic case where all particle masses are negligible, $p_f=p_i$
Then …

$$\frac{d\sigma}{d\Omega_{CM}} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

Feynman Diagrams

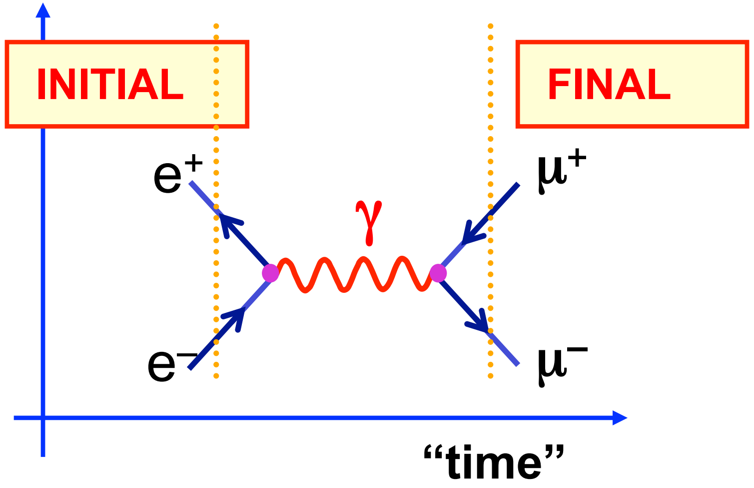
Particle interactions described in terms of Feynman diagrams





IMPORTANT POINTS TO REMEMBER:

- "time" runs from left – right, **only** in sense that:
  - LHS of diagram is initial state
  - RHS of diagram is final state
  - Middle is "how it happened"
- anti-particle arrows in –ve "time" direction

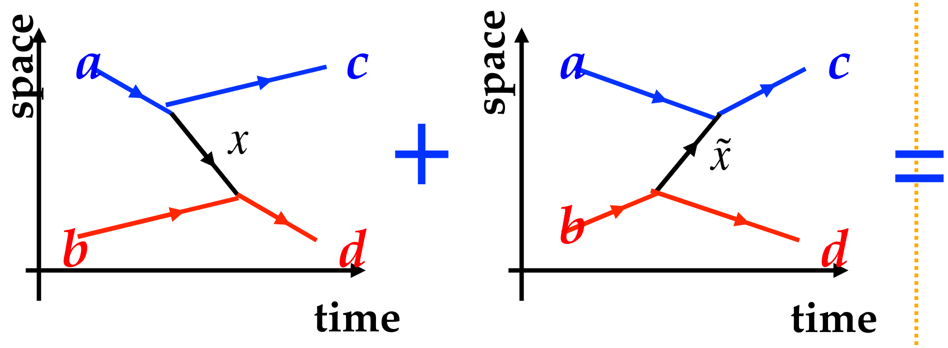


Virtual Particles

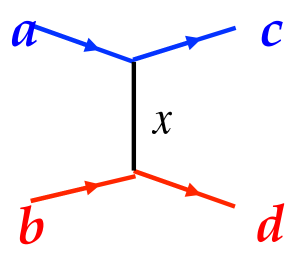
- In quantum field theory, Feynman diagrams correspond to sums of transitions ab→cd over all possible time-orderings and polarisations

- They are mathematical constructs that correspond to the concept of `virtual exchange particles' … the reality of which you may like to debate

"Time-ordered QM"



Feynman diagram



$$\frac{g_a g_b}{q^2 - m_x^2}$$

- **Momentum conserved at vertices**
- **Energy conserved overall, but not at individual vertices (uncertainty ppl)**
- **Exchanged particle "on mass shell"**

$$E_x^2 - |\vec{p}_x|^2 = m_x^2$$

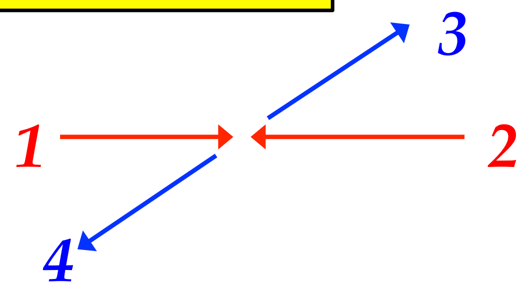
- **4-momentum (i.e. momentum AND energy) absolutely conserved at all interaction vertices**
- **Exchanged particle "off mass shell"**

$$E_x^2 - |\vec{p}_x|^2 = q^2 \neq m_x^2$$

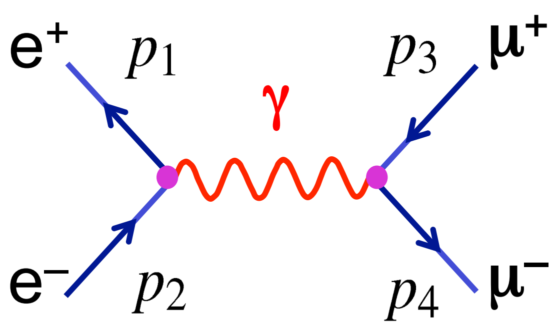
VIRTUAL PARTICLE

Mandelstam s, t and u

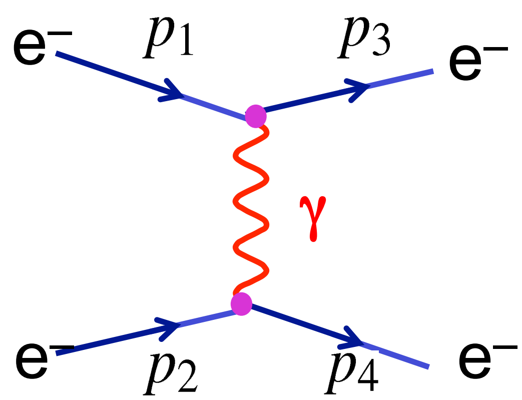
Scattering $1+2 \rightarrow 3+4$



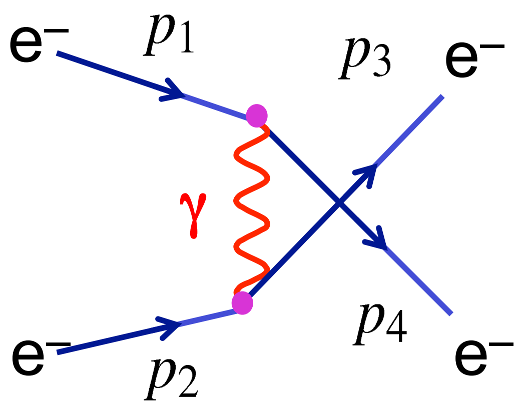
All diagrams with a single force carrier can be categorised according to the four-momentum of the exchanged particle as s, t or u channel:



s-channel



t-channel



u-channel

… u channel processes only occur for identical outgoing particles. They exist because the same final state can occur with two different exchange 4-momenta

Define kinematic variables: s, t and u

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2$$

s , t , u = squared 4-momentum of exchanged particle in corresponding s,t,u channel, though definitions are process independent

Manifestly Lorentz-invariant

Remembering Stanley Mandelstam 1928 - 2016

Stanley Mandelstam was born in Johannesburg, South Africa, on 12 December 1928 and brought up in the Natal Midlands, the son of a grocer who had recently emigrated from Latvia and a schoolteacher.

After he finished school, the family returned to Johannesburg and he embarked on a degree in chemical engineering at the University of the Witwatersrand.

The California Master Plan: exemplar of a modern higher education sector

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This was a vocational qualification, as his mother had wished, but Professor Mandelstam's real passion was mathematical physics. He therefore went on to a BA at the University of Cambridge (1954), followed by a PhD at the University of Birmingham (1956).

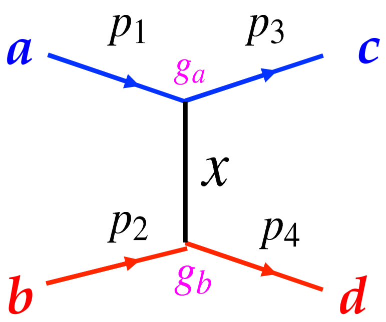
He continued his research at Birmingham for a further year and then moved to the US as a Boese fellow at Columbia University in New York (1957-58). He moved to the University of California, Berkeley as an assistant research physicist (1958-60) and was then appointed professor of mathematical physics at Birmingham (1960-63).

After that spell, however, Professor Mandelstam brought to an end his crossings of the Atlantic and settled for good at Berkeley as professor of theoretical physics. He was to remain there until he retired and became emeritus in 1994, although with short breaks as a *professeur associé* at Paris-Sud University.

Time-like and Space-like Virtual Particles

- **The four-momentum, q, carried by the (virtual) particle is determined from energy/momentum conservation at the vertices.**

… four-momentum transfer squared q^2 can be either positive or negative.

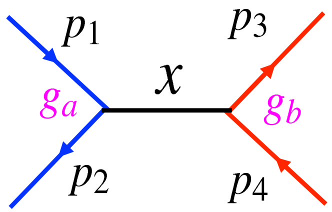


Here $q^2 = (p_1 - p_3)^2 = (p_4 - p_2)^2 = t$ **"t-channel"**

For elastic scattering: $p_1 = (E, \vec{p}_1); \quad p_3 = (E, \vec{p}_3)$

$$q^2 = (E - E)^2 - (\vec{p}_1 - \vec{p}_3)^2$$

$q^2 < 0$ **termed "space-like"**



Here $q^2 = (p_1 + p_2)^2 = (p_3 + p_4)^2 = s$ **"s-channel"**

In CoM: $p_1 = (E, \vec{p}); \quad p_2 = (E, -\vec{p})$

$$q^2 = (E + E)^2 - (\vec{p} - \vec{p})^2 = 4E^2$$

$q^2 > 0$ **termed "time-like"**

Feynman Rules for QED

- Quantum Electrodynamics (QED) is the quantum field theory of electromagnetism. See eg Feynman: `QED: The Strange Theory of Light and Matter'

- The matrix element for any QED process can be derived from a simple set of `Feynman Rules' starting from Feynman diagrams

- Rigorous derivation of Feynman rules is beyond scope of the course

Prescription:

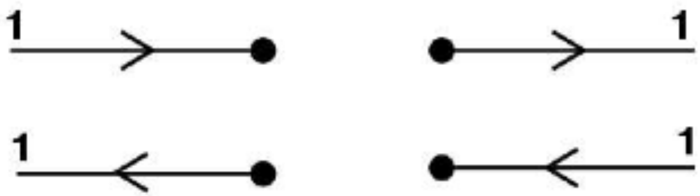
- Draw all Feynman diagrams that correspond to your process
- Apply Feynman rules to get corresponding mathematical expression
- The resulting expression is equal to $-iM_{fi}$ where M_{fi} is matrix element
- Add matrix elements if there are multiple diagrams (interference!)
- In general, insert sum into Fermi's Golden Rule
- For most cases we'll encounter, simply apply our result:

$$\frac{d\sigma}{d\Omega}_{cm} = \frac{1}{64\pi^2 s} \frac{p_f}{p_i} |M_{fi}|^2$$

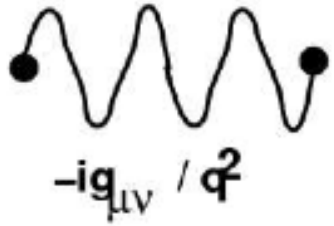
Feynman rules for QED of scalar particles + γ

- Scalar incoming / outgoing particles rather than spin-½ fermions, so not quite reality … but close (see Dirac equation to go to spin-½)
- Strictly, these rules only apply to `tree level' diagrams – i.e. no loops

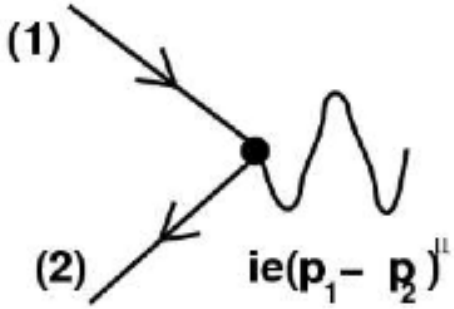
Incoming & outgoing particles



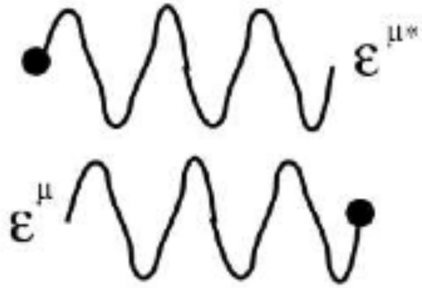
Exchange photon



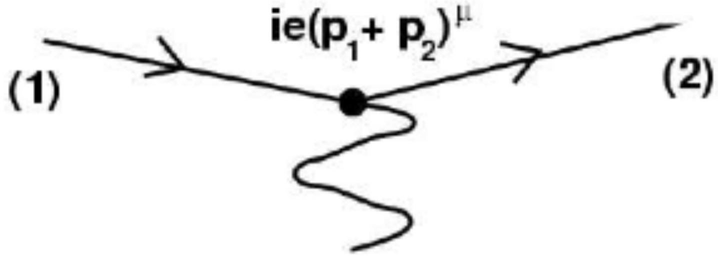
Particle - antiparticle QED vertex (s channel)



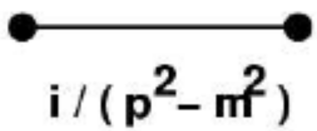
Initial or final state photon



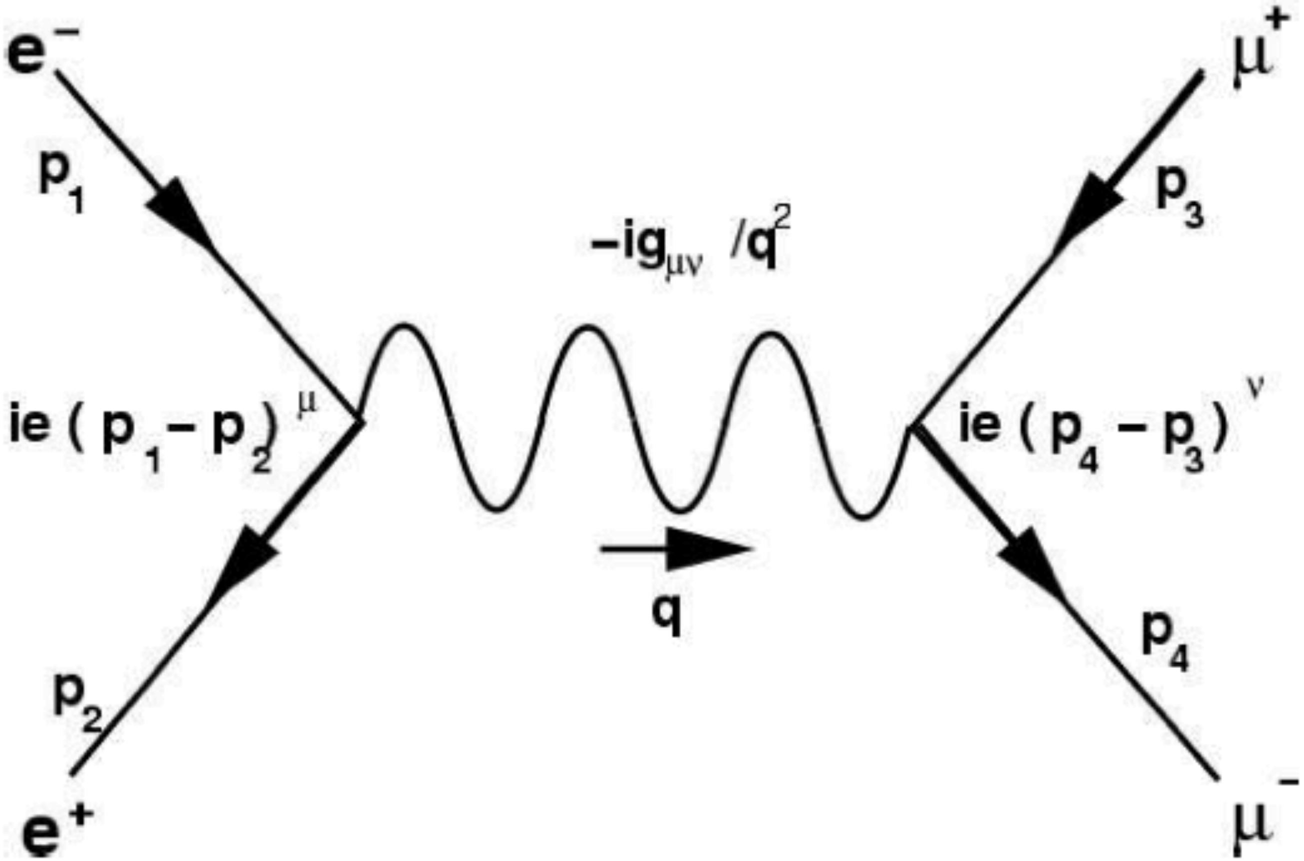
Particle - particle QED vertex (t,u channel)



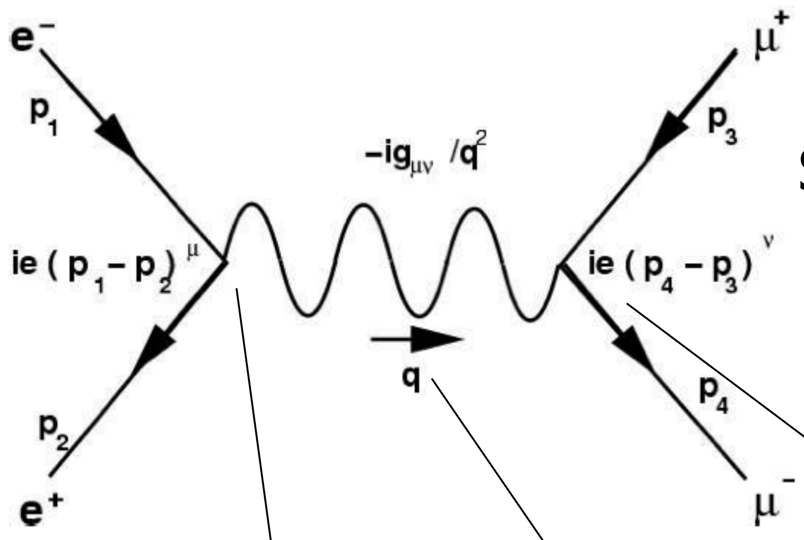
Exchange fermion



e.g. Spin-less e+e- →μ+μ-



Spin-less e+e- →μ+μ-



Single γ exchange

$$s = (p_1 + p_2)^2 = q^2$$

$$-iM_{fi} = ie(p_1 - p_2)^\mu(-ig_{\mu\nu}/q^2)ie(p_4 - p_3)^\nu$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s}\frac{(u-t)^2}{s^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s}\frac{(u-t)^2}{s^2} = \frac{e^4}{64\pi^2 s}\frac{(4p^2\cos\theta)^2}{(4p^2)^2} = \frac{\alpha^2}{4s}cos^2\theta$$

Cross-section decomposed

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} cos^2\theta$$

Coupling strength

Kinematic factor

Spin-dependent factor

One factor α for each vertex (due to a factor e in matrix element at each vertex)

Higher order terms go as α^n

➡ Diminish rapidly as n increases - perturbation expansion

Summary

- **Mandelstam variables and their manipulation**

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2$$

- **Feynman diagrams and their meaning, Feynman rules**

- **Example cross-section calculation for spin-less scattering with one photon exchange**

Next

More Feynman diagrams and cross-sections with examples
A simple argument to include spin=1/2