DEEP INELASTIC SCATTERING COURSE PROBLEMS

1. Explain what is meant by a fragmentation function. Let $D_i^{(h)}(z)$ be the fragmentation function for producing a hadron of type h with momentum fraction z in the fragmentation of a quark of type i. Recall the "master formula" of the QPM, and hence show that the normalized cross section for producing a hadron of type h in deep inelastic electron scattering is

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{(ep\to eh\ldots)}}{dt} = \frac{\sum e_i^2 f_i(x) D_i^{(h)}(z)}{\sum e_i^2 f_i(x)}.$$

Let

$$\frac{D_u^{\pi^+}(z)}{D_d^{\pi^+}(z)} = \eta(z).$$

Show that if valence quark scattering dominates, the ratio of π^+ to π^- production is of the form

$$\frac{N^{\pi^+}}{N^{\pi^-}} = \frac{4u(x)\eta(z) + d(x)}{4u(x) + d(x)\eta(z)}$$

[Note that by isospin invariance, $D_u^{\pi^+}(z) = D_d^{\pi^-}(z)$ and $D_u^{\pi^-}(z) = D_d^{\pi^+}(z)$.]

2. The cross section for Drell-Yan production can be written as

$$\frac{d\sigma}{dM^2} = \int dx_1 dx_2 \frac{d^2\sigma}{dx_1 dx_2}\delta(M^2 - sx_1x_2)$$

where

$$\frac{d^2\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9q^2}\sum_a e_a^2\left(q_a(x_1)\overline{q}_a(x_2) + \overline{q}_a(x_1)q_a(x_2)\right)dx_1dx_2.$$

Show that by rewriting the δ-function as $\delta(x_1x_2 - \tau)$, where $\tau = M^2/s = x_1x_2$, it is possible to rewrite this equation in the form

$$s^2\frac{d\sigma}{dM^2} = F(\tau)$$

What does this tell us about the energy dependence of the Drell-Yan cross section?

3. Use the Quark-Parton model to show that

$$\int_0^1 \frac{dx}{x}\left[F_2^{\nu n}(x) - F_2^{\nu p}(x)\right] = 2.$$

This is the *Adler* sum rule.

4. Explain how the cross section ratio

$$\frac{\sigma(\pi^+ C \to \mu^+ \mu^- X)}{\sigma(\pi^- C \to \mu^+ \mu^- X)}$$

can be used to test whether the $q\bar{q}$ fusion model of dimuon production works. Estimate the value of the ratio for $\tau = M^2/s$ small and for τ large.

[Note that C is an isoscalar target.]

5. Calculate the boundaries of the allowed kinematic region in the $\nu - Q^2$ plane for 300 GeV/c μp scattering. By calculating co-ordinate pairs or by using computer graphics, make a $\nu - Q^2$ diagram showing contours for muon scattering angles of $1°, 4°, 10°$ and $20°$ and for x_{Bj} values of 0.1, 0.3 and 0.5. What is the maximum allowed p_T in this reaction, and where does it fall in the $\nu - Q^2$ diagram?