Symmetries in Particle Physics

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Introduction

- In this course, we shall examine how certain types of symmetry can be expressed and applied in Particle Physics.
- Aim is to provide an intuitive approach, without going into rigorous proofs of the results we shall require.

Books

- Texts used for this course are
  - F. Halzen and A.D. Martin *Quarks and Leptons*, (Wiley, New York, 1984)
  - F.E. Close *An Introduction to Quarks and Partons* (Academic Press, New York, 1978)
  - W.M. Gibson and B.R. Pollard *Symmetry Principles in Elementary Particle Physics* (Cambridge University Press, 1984)
  - H.J. Lipkin *Lie Groups for Pedestrians* (North Holland, Amsterdam, 1965)

Groups

- (This lecture looks at material mainly in R.R. Stoll *Set Theory and Logic,* W.H. Freeman and Co., San Francisco, 1963)
- Purpose of lecture
  - To define a group
  - Investigate a few elementary properties of groups.

Definition of a Group

- A group is an ordered triple <*G*,•,*e*>, where
  - *G* is a set
  - • is a binary operation
  - *e* is a member of *G*
- 1.• is an associative operation: a•(b•c)=(a•b)•c
- 2. For each *a* in *G*, *e*•*a* = *a*
- 3. For each *a* in *G*, there exists an element *a'* in *G* such that *a'*•*a* = *e*

Group Properties

- e is a neutral element
  - *PROOF*
    - If a is in G, $\exists a'$ in G such that $a' \bullet a = e$
      and
      $\exists a''$ such that $a'' \bullet a' = e$
    - Then

$$
\begin{aligned}
a \bullet a' &= e \bullet (a \bullet a') = (a'' \bullet a') \bullet (a \bullet a') \\
&= a'' \bullet (a' \bullet a) \bullet a' = a'' \bullet (e \bullet a') \\
&= a'' \bullet a' = e
\end{aligned}
$$

    - So e must be a neutral element, since

$$a \bullet e = a \bullet (a' \bullet a) = (a \bullet a') \bullet a = e \bullet a = a$$

- Also a is *invertible,* since $a \bullet a' = a' \bullet a = e$

More Definitions

- If the elements in the group are finite then the group is **finite** and the number of elements is the **order** of the group
- If a group is not **finite**, it is **infinite**.
- If the operation satisfies $a \bullet b = b \bullet a$, then the group is **commutative** or **Abelian**.

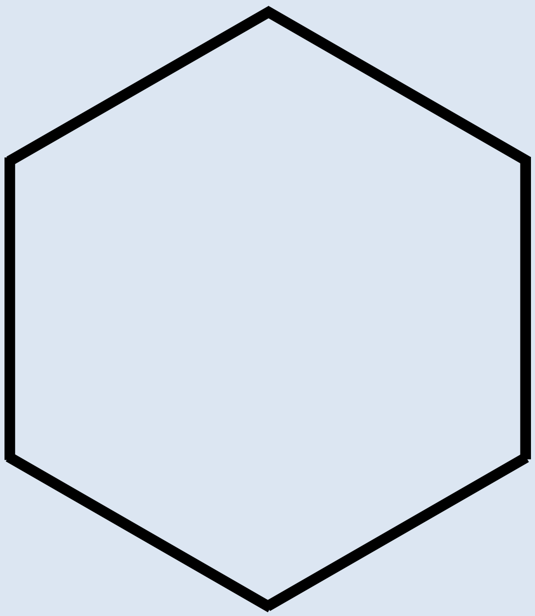
Examples

- **Addition**
  - Integers form a group under addition
  - Real numbers form a group under addition
  - Natural numbers?
  - What is the neutral element
- **Multiplication**
  - Real numbers form a group under multiplication
  - Integers?
  - What is the neutral element?
- Are the groups **Abelian** or **non-Abelian**?

More examples

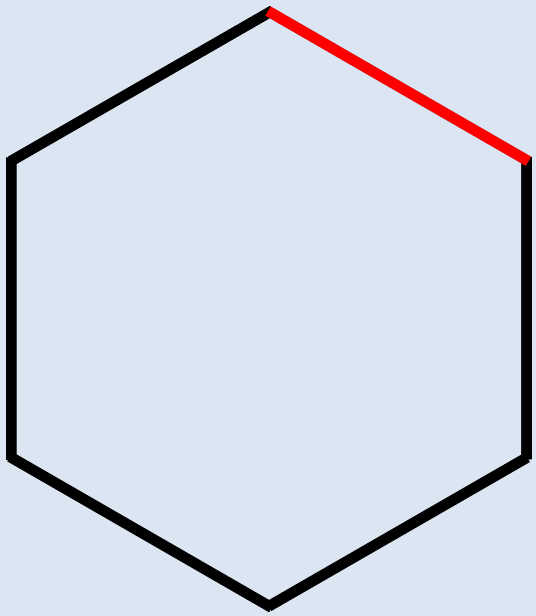
- **Matrices**
  - form an **Abelian** group under addition
  - form a ***non-Abelian*** group under multiplication
    - (**A**•**B** ≠ **B**•**A**)
- Most of the lectures will discuss transformations of different kinds (translations, rotations, etc.) and their associated groups.
- At first sight, this does not seem to fit the remit for a group, as transformations are monadic operators: **T***a*=*a'*.
- The theory of groups instead applies to how the transformations themselves are combined. (Non-Abelian groups yield more interesting structures.)

1-D Rotations



- In order to study rotational symmetry we need to define what it means to have an equivalent system
- Normally means that after the rotation the system is indistinguishable from the original
- We can keep track of this by artificially labelling a feature of the symmetric shape.
- Then when we perform an appropriate rotation, the system remains unchanged (*i.e.* invariant.)
- One dimensional rotations commute $R_A R_B = R_B R_A$

1-D Rotations



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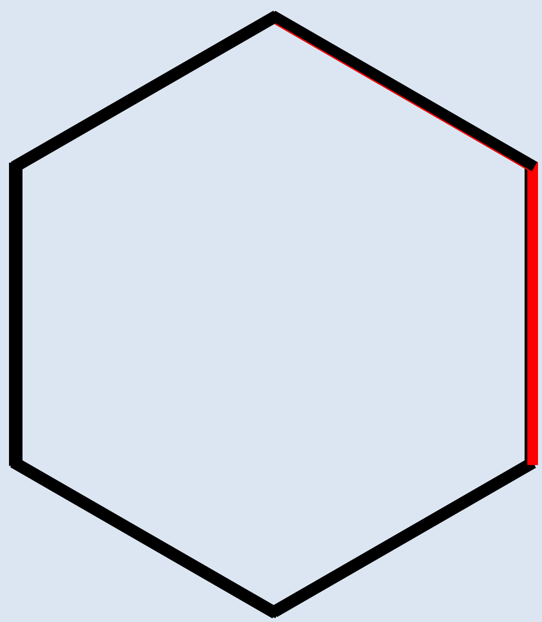

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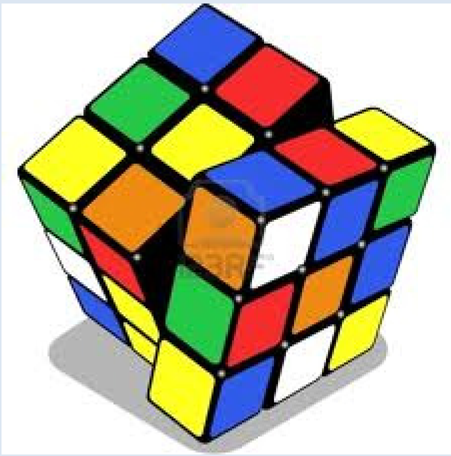
3-D Rotations



- When rotations can take place in more than one plane, order of rotations becomes important
  - Here $R_A R_B \neq R_B R_A$, in general.
  - There can be sub-groups with different properties. For example, a sub-class of 1D rotations inside the general 3D rotations clearly *does* commute.
  - Group structure should take this into account.

Quantum Mechanics

- In order to describe microscopic systems (such as systems of elementary particles) the transformations are described by quantum operators.
- Thus a typical operation would read

$$R|\psi\rangle = \lambda|\psi\rangle$$

and symmetries among operations would depend on whether

$$RQ|\psi\rangle = QR|\psi\rangle$$ or not.