SYMMETRY IN PARTICLE PHYSICS PROBLEMS

1. Using the defining relation for the Lie Algebra of angular momentum operators J_i and the results derived from this relation in Lecture 1, show that the eigenstates can have integer or half-integer values of j only.

   [Hint: define the highest and lowest eigenstates of J_3 as $|j\rangle$ and $|j-n\rangle$ respectively, and consider the effect of the ladder operators on these states.]
2. Define a rotation matrix for a spin-$\frac{1}{2}$ system which takes a spin directed along the $+y$ axis into the $+z$ axis.

   Using the usual representation of the Pauli matrices, the spinor

   $$\frac{1}{\sqrt{2}}\left\{\begin{pmatrix} i \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$$

   is an eigenstate of J_y with eigenvalue $+\frac{1}{2}\hbar$. Verify that it becomes an eigenstate of J_z with appropriate eigenvalue after the rotation.

3. The leptonic decay of a neutral vector meson proceeds via a virtual photon,

   $$V(q\overline{q}) \rightarrow \gamma^{\star} \rightarrow e^{+}e^{-}$$

   The *amplitude* for this process is proportional to the charge q of the quarks. By considering the SU(3) wavefunctions for the vector mesons, show that it follows that the leptonic decay widths of the vector mesons should be in the ratio

   $$\rho : \omega : \phi : \psi = 9 : 1 : 2 : 8$$

   neglecting mass effects. Use the tables provided to find the actual ratio of partial widths, and comment on the agreement. The quark wavefunctions for the ρ and the ω are $1/\sqrt{2}(u\overline{u} - d\overline{d})$ and $1/\sqrt{2}(u\overline{u} + d\overline{d})$ respectively.

4. Using the SU(6) wavefunctions:-

   (i) Verify that the neutron charge is zero in the Quark Model. Give the M,S and M,A contributions separately.

   (ii) Determine the magnetic moment of the Λ. Assume $\mu_s = \frac{3}{5}\mu_u$.

5. *(i)* Calculate the mixing angle in the 2^{++} nonet.

(*ii*) Use the quark model expression for meson masses to comment on why

$$m_{D^*} - m_D \ll m_{K^*} - m_K.$$

6. Classify the states Σ^+, Σ^-, $\Sigma^*(1385)^+$ and $\Sigma^*(1385)^-$ in terms of their U-spin quantum numbers. Explain why the photon can be regarded as a U-spin scalar, and hence explain why the transition

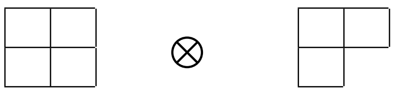
$$\Sigma^*(1385)^- \to \Sigma^- \gamma$$

is forbidden, while the transition

$$\Sigma^*(1385)^+ \to \Sigma^+ \gamma$$

is not.

7. Use the rules for Young's Tableaux to determine the expansion of



in SU(3), and interpret the result in terms of SU(3) multiplets.

8. Radiative transitions from vector mesons to pseudoscalar mesons are magnetic dipole transitions, *i.e.* the matrix element is given in the quark model by sandwiching the dot product of the magnetic dipole operator with the polarization vector of the emited photon between state vectors representing the initial and final state particles. Thus, for the transition $V \to P\gamma$, the matrix element is given by

$$M = \langle V|\vec{\mu}.\vec{\epsilon}|P\rangle$$

where

$$\vec{\mu} = \sum_{i=1}^{2} e_i \mu_i \vec{\sigma}_i,$$

and

$$\vec{\epsilon} = -\frac{1}{\sqrt{2}}(1, i, 0).$$

Use these definitions to show that

$$\frac{M(\rho^0 \to \pi^0\gamma)}{M(\omega \to \pi^0\gamma)} = \frac{1}{3}.$$