PARTICLE PHYSICS PROBLEMS

1. Evaluate the Clebsch-Gordon expansion of $\mathbf{10} \times \mathbf{8}$, using the method of Young Tableaux. For each of the supermultiplets you find, evaluate and draw its structure (occupancy of each weight) using the metod of weights.

2. Calculate the mixing angle in the 0^- multiplet. Suggest reasons why the mixing angle in this multiplet might be different from that of other multiplets.

3. The three dimensional harmonic oscillator Hamiltonian can be written as

$$H = \frac{1}{2}\hbar\omega \sum_{\mu=1}^{3} \left(a_\mu^\dagger a_\mu + a_\mu a_\mu^\dagger \right),$$

where $a_\mu^\dagger$, a_μ are *creation* and *annihilation* operators respectively, and

$$a_\mu = \left(\frac{m\omega}{2\hbar}\right)^{\frac{1}{2}} x_\mu + \frac{i}{(2m\omega\hbar)^{\frac{1}{2}}} p_\mu, \qquad \mu = 1, 2, 3.$$

Show that these operators satisfy the commutation relations

$$\begin{aligned} \left[a_\mu, a_\nu^\dagger\right] &= \delta_{\mu\nu}, \\ [a_\mu, a_\nu] &= \left[a_\mu^\dagger, a_\nu^\dagger\right] = 0. \end{aligned}$$

Defining linear combinations $a_\pm = (1/\sqrt{2})(a_1 \mp i a_2)$, derive the commutators $[a_\pm^\dagger, a_\pm]$ and $[a_\pm^\dagger, a_\mp]$ and hence show that the operators

$$\begin{aligned} \lambda_+ &= a_+^\dagger a_- \\ \lambda_- &= a_-^\dagger a_+ \\ \lambda_3 &= \frac{1}{2}(a_+^\dagger a_+ - a_-^\dagger a_-) \end{aligned}$$

where "+" and "-" have their usual meanings as linear combinations of operators λ_1, λ_2, are such that the $\lambda_i, i = 1, 3$ define a Lie Algebra.

4. In non-relativistic quantum mechanics the Schrödinger equation

$$H\psi(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t)$$

specifies the time development of ψ. Write down the equivalent time-reversed equation, i.e. with the transformation $t \to -t$. Consider the complex conjugate of this equation, and show that if $\psi(\mathbf{r}, t)$ satisfies the original Schrödinger equation then $\psi^*(\mathbf{r}, -t)$ satisfies the new equation. Use this to motivate the choice of

$$O_T \psi(\mathbf{r}, t) = \psi^*(\mathbf{r}, -t)$$

to describe the time reversal operator for spinless particles. Using this definition, show that

$$(O_T \chi, O_T \phi)_t = (\phi, \chi)_{-t} = (\chi, \phi)^*_{-t} \tag{1}$$

where the subscript t indicates the value of t for which the matrix element is evaluated. An operator with this property is called *antiunitary*. Write down the equivalent to equation (1) for a unitary operator.

Questions 1, 2 and 4 carry equal weight. Question 3 is rather lengthy and carries double marks.